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A model-referenced adaptive control system using a sensitivity model.

Desrosiers, Richard Albert

Monterey, California. Naval Postgraduate School

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THESIS

A MODEL-REFERENCED ADAPTIVE CONTROL

SYSTEM USING A SENSITIVITY MODEL

by

Richard Albert Desrosiers

June 1968

[redacted]

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SYSTEM USING A SENSITIVITY MODEL

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Richard Albert Desrosiers
Lieutenant, United States Navy
B. S., Western New England College, 1962

[redacted]

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ABSTRACT

Plant parameter variations due to environmental changes present a problem in any control system design. In this paper, two adaptive control techniques which compensate for large or small parameter variations are proposed. The variations in the parameters are identified by using the error between the plant output and a fixed model output together with the plant sensitivity functions which are used to identify plant parameter variations. One adaptive technique uses the identified parameter value to physically change the plant parameter and the other adaptive technique uses the identified parameter value to generate a compensating input to the plant. A mathematical model for the simultaneous generation of the desired output and the sensitivity functions is described. Several examples using both techniques are considered.

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1. Introduction.

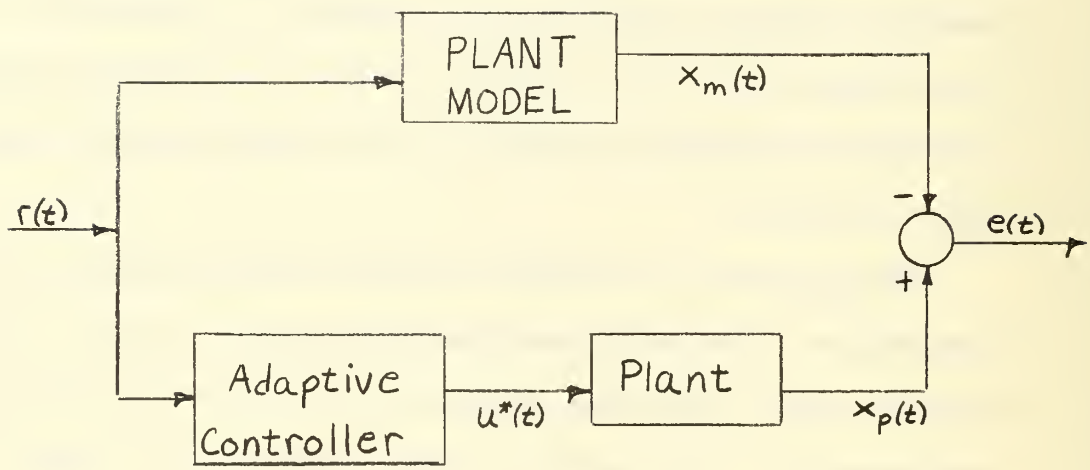
An adaptive control system must provide continuous information about the present state of the plant. It must identify any variations in the plant parameters or the plant dynamics; compare present system performance to the desired system performance; and modify the system to achieve overall optimum performance.

In the model-referenced, or model-following, adaptive control system of figure (1), the desired system performance is dictated by the plant model. When a parameter in the plant changes the output of the plant, $x_p(t)$, will differ from the output of the model, $x_m(t)$, thereby generating an error signal, $e(t)$. Once an error signal has been generated the adaptive system must identify which parameter, or parameters, of the plant have changed and how much they have changed. The adaptive controller must then use the information arrived at by the identification process to modify the plant.

The majority of the adaptive control systems presented in the literature consist of two major functional black boxes. [1], [2], [3], [4], [5]. One black box is called the identifier and the other is called the adaptive controller. The function of the identifier is to compute a performance measure, which is a functional relationship involving system characteristics in such a manner that operating conditions can be determined. The performance measure defines the optimum performance of the system. Identification is usually accomplished by deciding how the actual system performance relates to the desired system performance.

The adaptive controller is then used to calculate and implement either control signals, or controllable plant parameter changes to either maximize or minimize a performance measure. The optimum is generally approached gradually by a succession of decision, identification, and modification steps.

Model-Referenced Adaptive Control



r(t) - - input reference signal

$e(t) = x_p(t) - x_m(t)$

Fig. (1)

Bingulac, [6], uses sensitivity coefficients to measure the capability of the adjustable controller parameters to compensate for the variations of plant parameters. In this way it is possible to facilitate quick selection of the best possible controller parameters to be adjusted by the adaptive system. Dorf and Burzio, [7], use the sensitivity coefficients to directly control the variation of certain parameters of the plant so as to minimize the error signal in a model-referenced adaptive control system. In this paper the sensitivity functions are used to identify the amount of a parameter change. Thus, they form the basis of the idenfification process.

Two different adaptive techniques as discussed by Eveleigh, [5], are investigated in this paper. Technique number one is to identify what parameters have changed, determine how much they have changed, and then to re-adjust the plant parameters to new values as determined by the identification process. This assumes that the plant parameters are available for adjustment. Adjustment of the variable parameters of a plant by means of a characteristic equation which exhibits singular lines on the parameter plane is also described in this paper. Technique number two is to use the identified parameter changes to generate an optimum control, $u^*(t)$, which when applied to the plant, minimizes the error signal between the plant and the model.

This thesis discusses the implementation of these methods using a sensitivity model to identify the plant parameter changes. The use of a sensitivity model in an adaptive control system to perform the identification of parameter changes is a new concept and is proposed in this paper. Several examples involving digital computer simulation are included, the results illustrating the general effectiveness of the technique.

2. Parameter Identification.

In order for any adaptive technique to be successful, that is, for the plant output to follow the model output, and for the adaptive system to be practically realizable, the plant parameters must be accurately identified with easily obtained signals. The identification process of an adaptive control system is the most important and most critical process that the system performs.

The states of the plant, $\underline{x}_p$, are a function of time and the variable parameters of the plant, α_i .

$$\underline{x}_p = \underline{x}_p(t, \alpha_i)$$

Let α_{io} be the nominal or model value of the i-th parameter. The states of the plant can be expanded in a Taylor series about the nominal parameter value of the i-th parameter to get;

(1) $$\underline{x}_p(t,\alpha_i) = \underline{x}_p(t,\alpha_{io}) + \left.\frac{\partial \underline{x}_p(t,\alpha_i)}{\partial \alpha_i}\right|_{\alpha_i=\alpha_{io}} (\alpha_i - \alpha_{io}) + \frac{1}{2} \left.\frac{\partial^2 \underline{x}_p(t,\alpha_i)}{\partial \alpha_i^2}\right|_{\alpha_i=\alpha_{io}} (\alpha_i - \alpha_{io})^2 + \cdots$$

The Taylor series expansion of equation (1) gives an exact relationship for the states of the plant when its parameters vary about their nominal value.

The sensitivity of the states of the plant with respect to the i-th variable parameter is defined as;

(2) $$\underline{V}_i(t) \triangleq \left.\frac{\partial \underline{x}_p(t,\alpha_i)}{\partial \ln \alpha_i}\right|_{\alpha_i=\alpha_{io}}$$

which can be written as,

(3) $$V_i(t) = \alpha_{io} \frac{\partial \underline{x}_p(t,\alpha_{io})}{\partial \alpha_{io}}$$

If the parameter changes are small enough so that

$$\frac{(\alpha_i - \alpha_{io})^2}{2} \approx 0$$

then equation (1) becomes

(4)
$$\underline{x}_p(t,\alpha_i) = \underline{x}_p(t,\alpha_{io}) + \underline{v}_i(t) \frac{(\alpha_i - \alpha_{io})}{\alpha_{io}}$$

In equation (4), $\underline{x}_p(t, \alpha_{io})$ is the state vector of the plant with nominal parameter values. The state vector of the plant model, $\underline{x}_m(t, \alpha_{io})$, is the state vector of the same plant with invarient parameters set at the nominal values. Therefore, $\underline{x}_p(t, \alpha_{io}) = \underline{x}_m(t, \alpha_{io})$, and equation (4) becomes,

(5a)
$$\underline{e}(t) = \underline{v}_i(t) \frac{\Delta\alpha_i}{\alpha_{io}}$$

where

(5b)
$$\underline{e}(t) = \underline{x}_p(t,\alpha_i) - \underline{x}_m(t,\alpha_{io})$$

Equation (5b) when written in matrix form becomes

$$\begin{bmatrix} e_1(t) \\ e_2(t) \\ \cdot \\ e_n(t) \end{bmatrix} = \begin{bmatrix} x_{p1}(t) - x_{m1}(t) \\ x_{p2}(t) - x_{m2}(t) \\ \cdot \qquad \cdot \\ x_{pn}(t) - x_{mn}(t) \end{bmatrix}$$

where 1, 2,....,n refer to the states of the plant. Thus, it is obvious that equation (5a) can be written in scaler form as,

(6)
$$e(t) = v_i(t) \frac{\Delta\alpha_i}{\alpha_{io}}$$

where e(t) is the error between the output state of the plant and the output state of the model, and $v_i(t)$ is the sensitivity of the output of the plant with respect to the i-th variable parameter.

If there are N independent variable parameters in the plant, a Taylor series expansion is made for each parameter similar to equation (1) and the N expansions are added by the laws of superposition to get,

$$e(t) = \sum_{i=1}^{N} v_i(t) \frac{\Delta \alpha_i}{\alpha_{io}} \tag{7}$$

Equation (6) tells us that the error between the output of the plant and the output of the model which is caused by a parameter variation is directly proportional to the sensitivity of the output of the plant. This statement on first glance appears to be very simple, and is indeed logical since the sensitivity functions give an indication of how the output of the plant will vary when a parameter is changed. However, equation (6) implies a few subtle points which are best illustrated by the following example.

Assume that the plant under consideration is first order and that it is driven by a unit impulse function. The response or output of the plant is given by;

$$\frac{dx}{dt} + \alpha(t)x(t) = \delta(t) \tag{8}$$

where $\alpha(t)$ is the variable parameter of the plant and $\delta(t)$ is the impulsive input. The solution to equation (8) can be written as;

$$x(t) = e^{-\int_0^T \alpha(t)dt} \tag{9}$$

If a step change in the parameter occurs at time t = T, then the time variation of the parameter is written as;

(10) $$\alpha(t) = \alpha(o)\, u(t) + \Delta\alpha(T)\, u(t-T)$$

Integrating both sides of equation (10) for the time interval $0 \leq t \leq t$ yields;

(11) $$\int_0^T \alpha(t)\,dt = \alpha(o)\, t\, u(t) + \Delta\alpha(T)(t-T)\, u(t-T)$$

From equation (11), the solution to equation (8) becomes;

(12) $$x(t) = e^{-\alpha_o t}\, e^{-\Delta\alpha(T)(t-T)\,u(t-T)}$$

If the change in the parameter at time T is small enough to allow the exponential to be approximate by the first two terms of its Taylor series expansion, then;

(13) $$x(t) = e^{-\alpha_o t}\left[1-(t-T)\,\Delta\alpha(T)\right]$$

Now, if the desired response, or the parameter invarient response, is given by

$$x_o(t) = e^{-\alpha_o t}$$

then the partial derivative of $x_o(t)$ with respect to the nominal parameter value is given by;

(14) $$\frac{\partial x_o(t)}{\partial \alpha_o} = -t\, e^{-\alpha_o t}$$

which when time shifted to $t = T$ becomes;

(15) $$\frac{\partial x_o(t-T)}{\partial \alpha_o} = -(t-T)\, e^{-\alpha_o (t-T)}$$

Equation (13) then becomes;

$$x(t,\alpha) = e^{-\alpha_0 t} - (t-T)\,\Delta\alpha(T)\,e^{-\alpha_0 t}$$

$$= x_0(t,\alpha_0) + e^{-\alpha_0 T}\,\frac{\partial x_0(t-T)}{\partial \alpha_0}\,\Delta\alpha(T)$$

(16) $$x(t,\alpha) = x_0(t,\alpha_0) + x_0(T,\alpha_0)\,\Delta\alpha(T)\,\frac{\partial x_0(t-T)}{\partial \alpha_0}$$

In a model-referenced adaptive control system, the error signal is given by,

$$e(t) = x(t,\alpha) - x_0(t,\alpha_0)$$

Therefore, for a parameter change occurring at time t = T equation (16) becomes;

(17) $$e(t-T) = x_0(T,\alpha_0)\,\Delta\alpha(T)\,\frac{\partial x_0(t-T)}{\partial \alpha_0}$$

which with the use of equation (2) becomes;

(18) $$e(t-T) = x_0(T,\alpha_0)\,\frac{\Delta\alpha(T)}{\alpha_0}\,v(t-T)$$

where $v(t-T)$ is the sensitivity of $x_o(T, \alpha_0)$ with respect to α_0 .

Equation (18) shows that the error signal is dependent upon the value of the desired response at the time of the parameter change and upon the sensitivity of the desired response with respect to the nominal parameter value. In order to use the sensitivity functions, $v_i(t)$, in the identification process, the following criteria must exist:

1. The parameter changes must be approximately less than fifteen percent for an error less than five tenths of a percent.
2. The sensitivity functions must be generated from the output of a parameter invarient model.

3. The generation of the sensitivity functions must not begin until a parameter change occurs.

4. If there exists more than one variable parameter in the plant, all parameters must be independent of each other.

It will be computationally convenient to take the derivative of equation (5) with respect to time to get,

(19) $$\dot{e}(t) = \dot{V}_i(t)\,\frac{\Delta\alpha_i}{\alpha_{io}} + V_i(t)\,\frac{d(\Delta\alpha_i)}{dt}\cdot\frac{1}{\alpha_{io}}$$

The plant parameters are assumed to change in a piece-wise,constant, step-like manner. Therefore, over a small time interval Δt, the parameter change is constant and

$$\frac{d(\Delta\alpha_i)}{dt} \equiv 0$$

Over this small time interval, $\dot{e}(t)$ and $\dot{v}(t)$ can be approximated by the slope of their respective curves. Thus equation (19) becomes,

$$\frac{e(t+\Delta t) - e(t)}{\Delta t} = \frac{V_i(t+\Delta t) - V_i(t)}{\Delta t}\cdot\frac{\Delta\alpha_i}{\alpha_{io}}$$

which reduces to,

(20) $$[e(t+\Delta t) - e(t)] = [V_i(t+\Delta t) - V_i(t)]\,\frac{\Delta\alpha_i}{\alpha_{io}}$$

Therefore, the change in parameter α_i , for the case where N = 1, becomes,

(21) $$\Delta\alpha_1 = \frac{[e(t+\Delta t) - e(t)]}{[V_1(t+\Delta t) - V_1(t)]}\cdot\alpha_{io}$$

Prior to the occurrence of a parameter change, the identification process is not necessary. At the time t = T, the time the first parameter change occurs, e(t) = e(T) and v(t) = v(T). Therefore, the amount that the parameter α_1 changes is given by,

$$\Delta\alpha_1 = \alpha_{10} \frac{e(T+\Delta t) - e(T)}{V_1(T+\Delta t) - V_1(T)} \tag{22}$$

For a plant which has only one variable parameter, the amount of the parameter change can be identified by sampling the error signal and the sensitivity signal at time t = T + Δ t and applying equation (22).

If the plant has N independent variable parameters, equation (20) becomes,

$$\begin{aligned}[e(t+\Delta t) - e(t)] = [V_1(t+\Delta t) - V_1(t)]C_1 + [V_2(t+\Delta t) - V_2(t)]C_2 + \cdots \\ \cdots [V_N(t+\Delta t) - V_N(t)]C_N\end{aligned} \tag{23}$$

where $$C_i = \frac{\Delta\alpha_i}{\alpha_{i0}}$$

Equation (23) contains N unknowns. For a one parameter system it was necessary to sample the error signal and the sensitivity signal only once to determine the amount of the parameter change. For a system with N variable parameters it is necessary to solve N simultaneous equations to determine N parameter changes. Therefore, it is necessary to sample the error and sensitivity signals N times.

$$\begin{aligned}
[e(T+\Delta t) - e(T)] &= [V_1(T+\Delta t) - V_1(T)]C_1 + \cdots + [V_N(T+\Delta t) - V_N(T)]C_N \\
[e(T+2\Delta t) - e(T+\Delta t)] &= [V_1(T+2\Delta t) - V_1(T+\Delta t)]C_1 + \cdots + [V_N(T+2\Delta t) - V_N(T+\Delta t)]C_N \\
&\vdots \qquad\qquad\qquad\qquad\qquad \vdots \\
[e(T+N\Delta t) - e(T+(N-1)\Delta t)] &= [V_1(T+N\Delta t) - V_1(T+(N-1)\Delta t)]C_1 + \cdots
\end{aligned} \tag{24}$$

Equations (24) represent N simultaneous equations arrived at by sampling N times. These equations can be written in matrix form as,

(25) $$E = VC$$

where E is an Nx1 vector

V is an NxN matrix and $|V| \neq 0$

C is an Nx1 vector

Now:

(26) $$C = V^{-1}E$$

from which the parameter changes can be determined. That is,

(27) $$\Delta\alpha_i = \alpha_{io} C(i,1)$$

3. Sensitivity Functions.

The previous section showed how the sensitivity functions can be used in the identification of variable plant parameters. In this section the method of generating sensitivity functions is discussed. The sensitivity functions give an indication of how the states of a plant will vary when its parameters change. As a result, each sensitivity function can be considered as an error in the plant output caused by a parameter change. The total error in the plant's output is a linear sum of the individual errors, or sensitivity functions, as was shown in equation (7).

The sensitivity functions were previously defined as,

$$\underline{V}_i(t) \triangleq \alpha_{io} \frac{\partial \underline{X}(t,\alpha_{io})}{\partial \alpha_{io}} \tag{28}$$

Consider the linear time varying system

$$\dot{\underline{X}}_p(t,\alpha) = A(t,\alpha)\,\underline{X}_p(t,\alpha) + B(t)\,\underline{u}(t,\alpha) \tag{29}$$

Taking the partial derivative of equation (29) with respect to α and evaluating at $\alpha = \alpha_0$ yields,

$$\left.\frac{\partial \dot{\underline{X}}_p(t,\alpha)}{\partial \alpha}\right|_{\alpha=\alpha_0} = A(t,\alpha)\left.\frac{\partial \underline{X}_p(t,\alpha)}{\partial \alpha}\right|_{\alpha=\alpha_0} + \underline{X}_p(t,\alpha)\left.\frac{\partial A(t,\alpha)}{\partial \alpha}\right|_{\alpha=\alpha_0} + B(t)\left.\frac{\partial u(t,\alpha)}{\partial \alpha}\right|_{\alpha=\alpha_0} \tag{30}$$

Substituting (28) into (30) yields,

$$\dot{\underline{V}}(t) = A(t,\alpha_0)\,\underline{V}(t) + \alpha_0 C(t)\,\underline{X}_p(t,\alpha_0) + \alpha_0 B(t)\left.\frac{\partial \underline{u}(t,\alpha)}{\partial \alpha}\right|_{\alpha=\alpha_0} \tag{31}$$

where

$$C_{ij}(t) = \left.\frac{\partial A_{ij}(t,\alpha)}{\partial \alpha}\right|_{\alpha=\alpha_0}$$

H. D'Angelo, [8], proposed using the plant sensitivity functions and all of their derivatives in a performance measure and then minimizing the performance measure with respect to the control, u(t). The optimum control which resulted was a function of the sum of the sensitivity function and its derivatives, that is

$$v_i(t) + \frac{\partial v_i(t)}{\partial \alpha_i} + \frac{\partial^2 v_i(t)}{\partial \alpha_i^2} + \cdots$$

S. Kahne, [9], proposed using sensitivity functions in a performance measure also. Kahne's approach was to write a sensitivity state equation similar to (31), from which he generated the sensitivity functions as shown in figure (2).

The approach taken in this paper is to use the sensitivity functions to identify plant parameter changes. In this way the sensitivity functions can be used as reference signals and must not vary when the plant parameters vary.

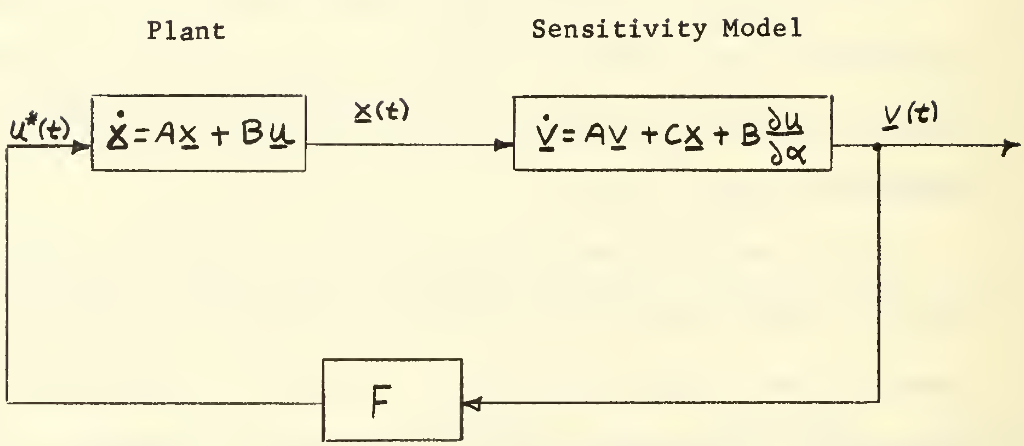
In the method of generating sensitivity functions shown in figure (2), if a plant parameter changes, then x(t), v(t), and u(t) will change. Thus, the sensitivity functions will not give a good measure of how the plant states vary with respect to some nominal parameter, and will not be useful for the identification process.

Therefore, in order for the sensitivity functions to be useful in the identification process of the adaptive control system, they must not be dependent on the states of the plant or upon the control which drives the plant.

If the sensitivity functions were generated by a plant model which is invariant to parameter changes, then the sensitivity functions could be used for parameter identification. Equation (19) reduces to

(32) $$\dot{\underline{v}}(t) = A(t,\alpha_0)\,\underline{v}(t) + \alpha_0 C(t)\,\underline{x}_m(t,\alpha_0)$$

Kahne's Plant and Sensitivity Model



u*(t) is the optimum control which is generated by the process in block F

Fig. (2)

Equation (32) represents a mathematical model which can be used to generate the sensitivity functions. It should be noted that,

$$\frac{\partial u(t)}{\partial \alpha} \equiv 0$$

Kahne and D'Angelo penalized their system for having variable parameters by including the sensitivity functions in their cost functions. The approach taken in this paper is not to include a penalty for sensitivity, but to use sensitivity for the identification problem.

4. Model.

The adaptive control system must be able to identify the plant parameter changes from signals which are easily generated. The purpose of the plant model is to generate the desired output and the sensitivity of the output with respect to the variable parameters. The model must generate the desired output and sensitivity signals simultaneously so that there will be a minimum time delay in correcting for the parameter changes. Rajko Tomovic, [10], has presented a method whereby the sensitivity functions can be generated simultaneously with the desired states. This method is illustrated in figure (3).

Figure (3) shows that the response of the plant model, $x(s,q_1,q_2)$, can be introduced into an identical model to obtain the sensitivity functions, v_1 and v_2. The sensitivity functions can be generated on an analog or digital computer using the state variable approach. This paper employs the latter method for reasons which will become apparent later.

It should be re-emphasized that the sensitivity functions should not be generated until a parameter change has occurred. The sensitivity model should be triggered when $|e(t)| > 0$ or when $t = T$. Where T is the time at which the first parameter change occurs.

Example (1)

This example serves to illustrate Tomovic's method of generating sensitivity functions. Consider the first order system of figure (4a).

Let $y(t) = E$ for $0 \leq t \leq t_f$

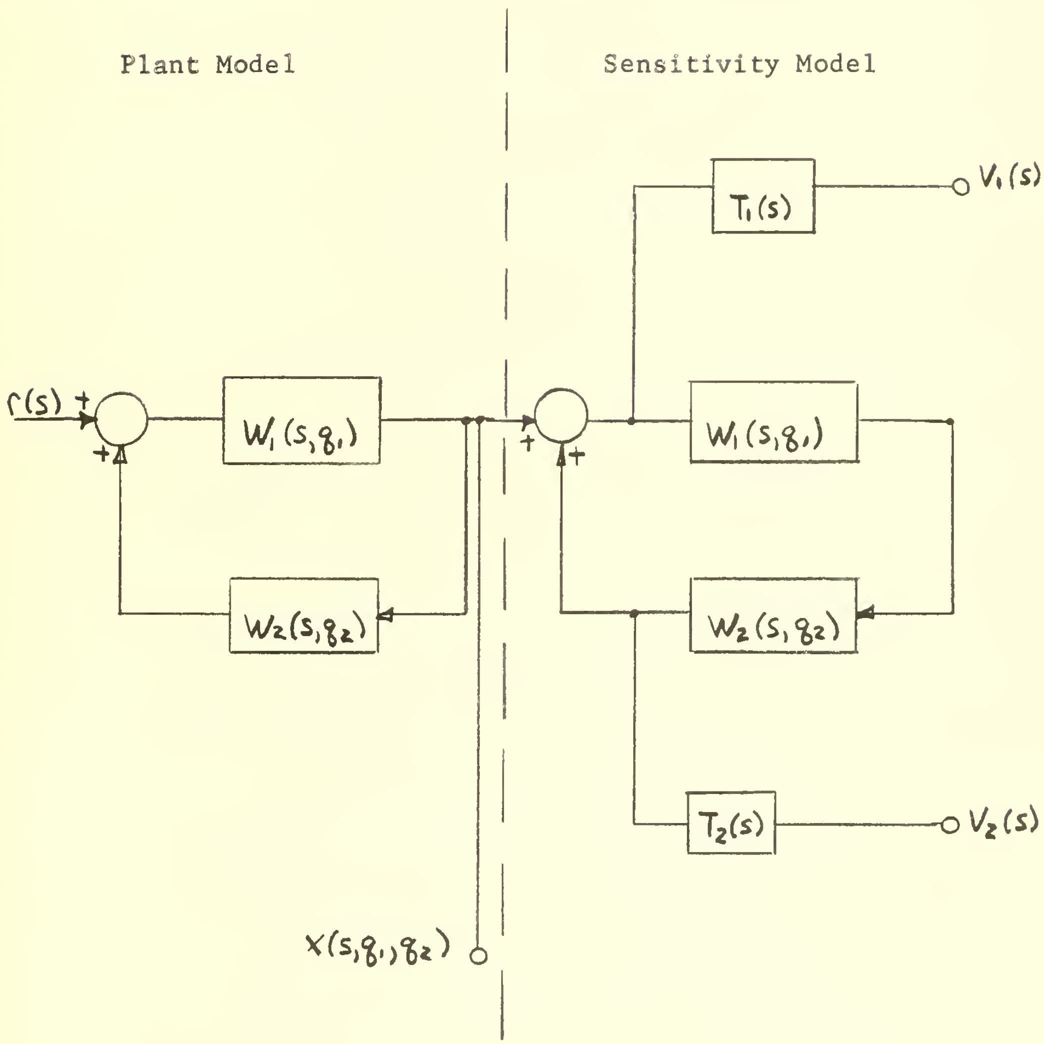
therefore $y(s) = E/s$. From figure (4a) it follows that

$$x(s) = \frac{E}{s(s+\alpha_0)} \qquad \text{which gives} \qquad x(t) = E\left(\frac{1}{\alpha_0} - \frac{1}{\alpha_0}e^{-\alpha_0 t}\right)$$

where $\alpha_0 = A_0 + K_0$

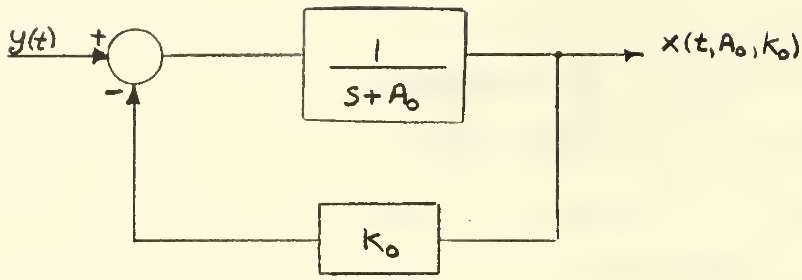
Tomovic's Plant and Sensitivity Model

$$T_i(s) = \frac{q_i}{W_i(s)} \frac{\partial W_i(s)}{\partial q_i}$$



q_1 and q_2 are the variable plant parameters

Fig. (3)


y(t)
+
-
1
S+A₀
x(t,A₀,K₀)
K₀

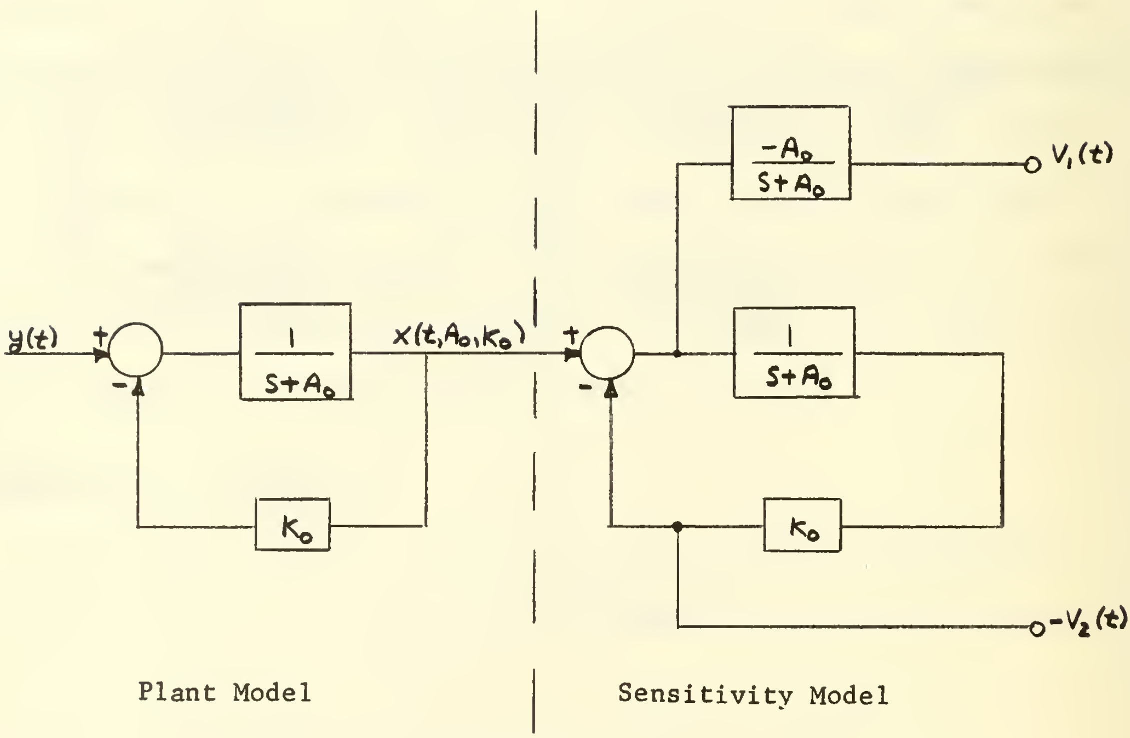

y(t)
+
-
1
S+A₀
x(t,A₀,K₀)
K₀
+
-
-A₀
S+A₀
V₁(t)
1
S+A₀
K₀
-V₂(t)
Plant Model
Sensitivity Model

Fig. (4b)

From equation (28)

$$V_1(t) = A_o \frac{\partial x(t, A_o, K_o)}{\partial A_o}$$

$$V_2(t) = K_o \frac{\partial x(t, A_o, K_o)}{\partial K_o}$$

which yields,

$$V_1(t) = E A_o \left[-\frac{1}{\alpha_o^2} + \left(\frac{t}{\alpha_o} + \frac{1}{\alpha_o^2} \right) e^{-\alpha_o t} \right]$$

$$V_2(t) = E K_o \left[-\frac{1}{\alpha_o^2} + \left(\frac{t}{\alpha_o} + \frac{1}{\alpha_o^2} \right) e^{-\alpha_o t} \right]$$

Tomovic's method is shown in figure (4b). From figure (4b) it follows that;

$$\left[X(s) + V_2(s) \right] \frac{K_o}{s + A_o} = -V_2(s) \quad \text{or} \qquad V_2(s) = \frac{-E K_o}{s(s+\alpha_o)^2}$$

which yields

$$V_2(t) = E K_o \left[\frac{-1}{\alpha_o^2} + \left(\frac{t}{\alpha_o} + \frac{1}{\alpha_o^2} \right) e^{-\alpha_o t} \right]$$

and

$$V_1(s) = \frac{-A_o E}{s(s+\alpha_o)^2}$$

which yields

$$V_1(t) = E A_o \left[-\frac{1}{\alpha_o^2} + \left(\frac{t}{\alpha_o} + \frac{1}{\alpha_o^2} \right) e^{-\alpha_o t} \right]$$

Thus, Tomovic's method yields the same equations for the sensitivity functions as were found by direct differentiation.

The following state equations can be used on a digital computer as the plant and sensitivity models.

$$\dot{x}(t) = -(A_0 + K_0)\,x(t) + y(t)$$

$$\dot{V}_1(t) = -A_0\,x(t) - A_0 V_1(t) - A_0 V_2(t)$$

$$\dot{V}_2(t) = -K_0\,x(t) - (A_0 + K_0)\,V_2(t)$$

These state equations can be solved to determine the desired output and the sensitivity functions for use in the identification process. The proposed model for the adaptive control system is a mathematical model which is independent of parameter variations and can be simulated on a digital controller.

5. Illustrations of the Parameter Identification Process.

Example (1) First order plant, one variable parameter

This example serves to illustrate the effectiveness of the parameter identification process on a plant which has only one variable parameter. The sensitivity functions are generated by the Tomovic method discussed previously. The system shown in figure (5) was simulated on an IBM-360 digital computer using the following state equations:

$$\dot{X}_m(t) = -\alpha_o X_m(t) + \alpha_o E(t)$$

$$\dot{X}_p(t) = -\alpha X_m(t) + \alpha E(t)$$

$$\dot{V}(t) = \dot{X}_m(t) - \alpha_o V(t)$$

At time t = T, parameter α underwent a step change by an amount $\Delta\alpha$ thus creating a non-zero error which initiates the generation of the sensitivity functions. The sensitivity functions and the error signal are sampled at time $t = T + \Delta t$. The amount of the parameter change which is determined by the identification process is given by;

$$\Delta\alpha_I(t) = \alpha_o \frac{e(T+\Delta t) - e(T)}{V(T+\Delta t) - V(T)}$$

where α_o = nominal parameter value= 0.20

Δt = sampling interval = 0.10 sec.

The new value of the parameter, as determined by the identification process, is given by;

$$\alpha_I(t) = \alpha_o + \Delta\alpha_I(t)$$

The percent error in the identification process is defined as;

$$\%\ \text{Error} \triangleq \frac{|\alpha(t) - \alpha_I(t)|}{\alpha(t)} \times 100\ \%$$

One Variable Parameter Identification

Example, Step Parameter Change

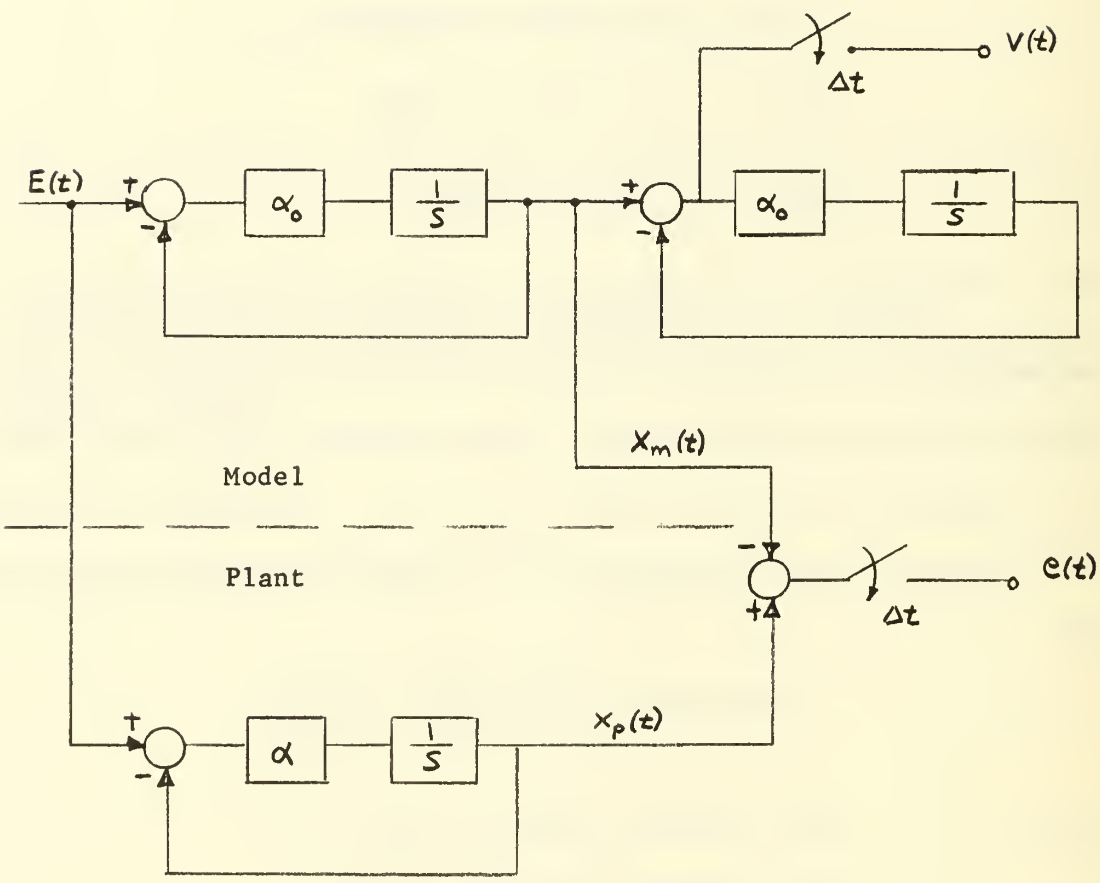


Fig. (5)

where α (t) is the actual value of the plant parameter after the change occurs. The percent error in the identification process is a measure of the accuracy of the process.

Table (I) shows the results of applying a step change of $\Delta\alpha$ = -0.01 to the parameter at T = 1.5 sec. The reference input for this example, E(t), is a step function of magnitude ten.

Table (I)

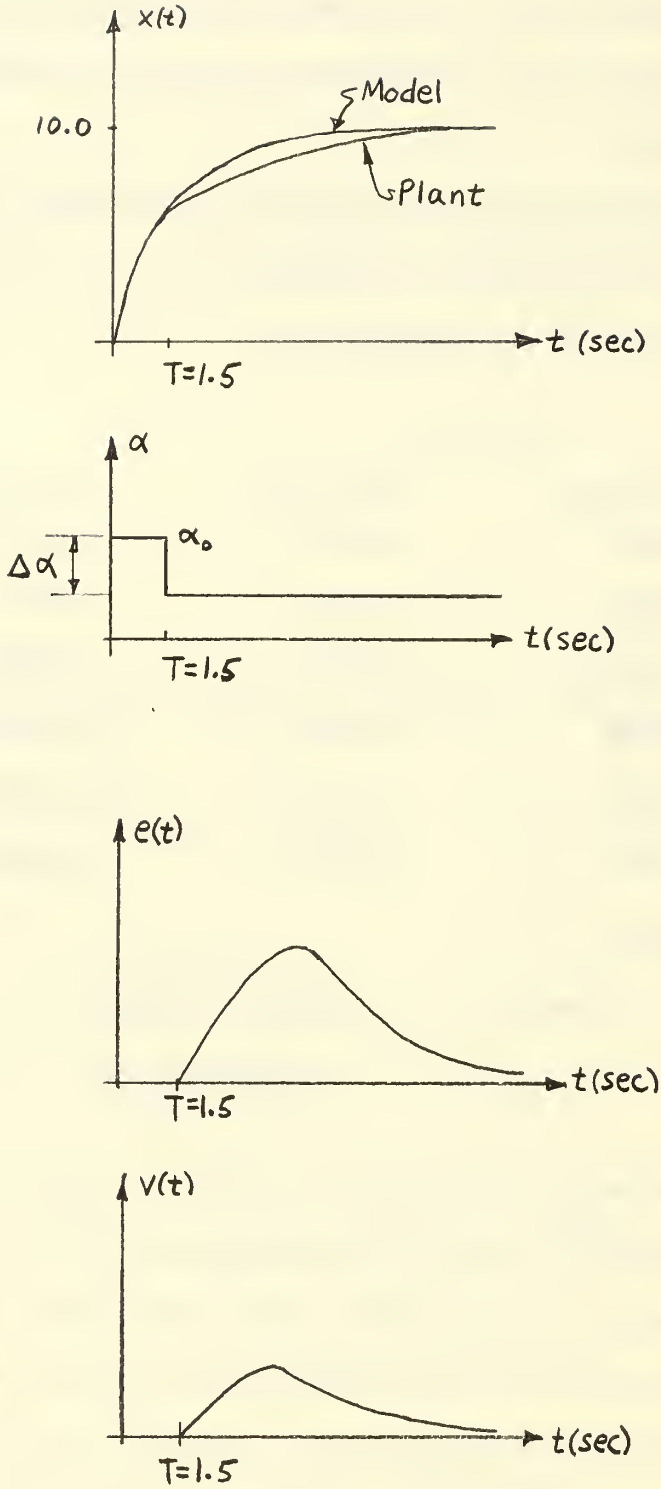
Time	identified $\alpha_I(t)$	identified $\Delta\alpha_I(t)$	% error in identification
1.60	0.19000	-0.01000	0.0000
1.70	0.18999	-0.01001	0.005270
1.80	0.18998	-0.01002	0.010729
1.90	0.18997	-0.01003	0.016281
2.00	0.18996	-0.01004	0.021991

It should be noted that;

$$\Delta\alpha_I(1.6) = \alpha_o \frac{e(1.6) - e(1.5)}{V(1.6) - V(1.5)}$$

$$\Delta\alpha_I(1.7) = \alpha_o \frac{e(1.7) - e(1.6)}{V(1.7) - V(1.6)}$$

Figure (6b) shows the nature of the parameter change; (6c) shows the error between the output of the plant and the output of the model; and (6d) shows the sensitivity of the model output with respect to the nominal parameter. The accuracy of the identification process, as indicated by the percent error in the identification process, can be seen to decrease as time increases. This is primarily due to the build up of truncation and round-off errors in the computer simulation. The continuous identification process is accomplished by sampling the error signal and the sensitivity signal every Δ t seconds and performing the identification process at the end of each sampling interval. Thus, the errors are allowed to build up from sample to sample.



(a) Plant and model outputs

(b) Variable Parameter α

(c) Error between plant and model

(d) Sensitivity of output with respect to α

Fig. (6)

Several runs were made with a step reference input and a step parameter change. In each run, a different parameter change occurred. These parameter changes occurred at different times. The percent error in the identification process was seen to increase as the amount of the parameter change increased. The accuracy of the method employed to determine parameter changes was seen to be very good for changes up to about fifteen percent.

Example (2)

This example is the same as example (1) with the exception of the reference input and the amount of the parameter change. The reference input is given by;

$$E(t) = 10.0 \sin (t)$$

The variable parameter was changed from $\alpha = 0.20$ to $\alpha = 0.21$ at time $T = 1.5$ seconds. The results of this example are shown in table (II).

Table (II)

Time	identified $\alpha_I(t)$	identified $\Delta\alpha_I(t)$	% error in identification
1.60	0.21000	0.01000	0.000000
1.70	0.20999	0.00990	0.0047686
1.80	0.20998	0.009979	0.0097922
1.90	0.20997	0.009968	0.015157
2.00	0.20996	0.009956	0.021089

Example (1) and (2) have illustrated the accuracy of the identification process on a plant which is of first order and has only one variable parameter. The identification process can easily be accomplished by the adaptive control system. It should be noted that self-adaptation was not attempted in either example.

Example (3) Second order plant, one variable parameter

In this example a second order plant with one variable parameter undergoes a ramp change in the parameter. That is, the parameter is a constant value until it undergoes a time varying change at time t = T.

The plant which is used in this example is shown in figure (7a). The nominal value of the variable parameter is $\alpha_0 = 4.00$. At T = 1.00 seconds the parameter is changed to

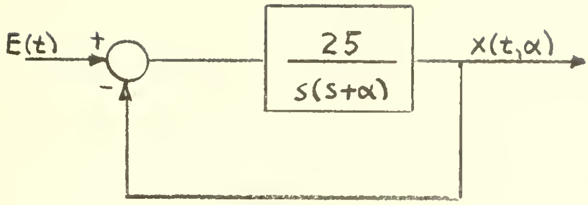
$$\alpha(t) = 4.0 + 0.1\ t$$

as shown in figure (7b). The amount of the parameter change is identified in the same manner as was discussed in example (1). The sampling time used is $\Delta t = 0.01$ sec. A step reference input of magnitude ten is used. Table (III) gives the results of the computer simulation of this example and the value of the parameters which are determined by the identification process are plotted as crosses on figure (7b). The accuracy of the identification process is given by the percent error in the identification which was defined in example (1).

Table (III)

Time	actual $\alpha(t)$	identified $\alpha_I(t)$	% error in identification
1.01	4.10100	4.10089	0.00465
1.02	4.10200	4.10178	0.00532
1.03	4.10300	4.10281	0.004625
1.04	4.10400	4.10390	0.002254
1.05	4.10500	4.10508	0.00209
1.06	4.10600	4.10636	0.00800

Figure (7b) shows that the parameter values which are determined by the identification process follow the actual parameter values very

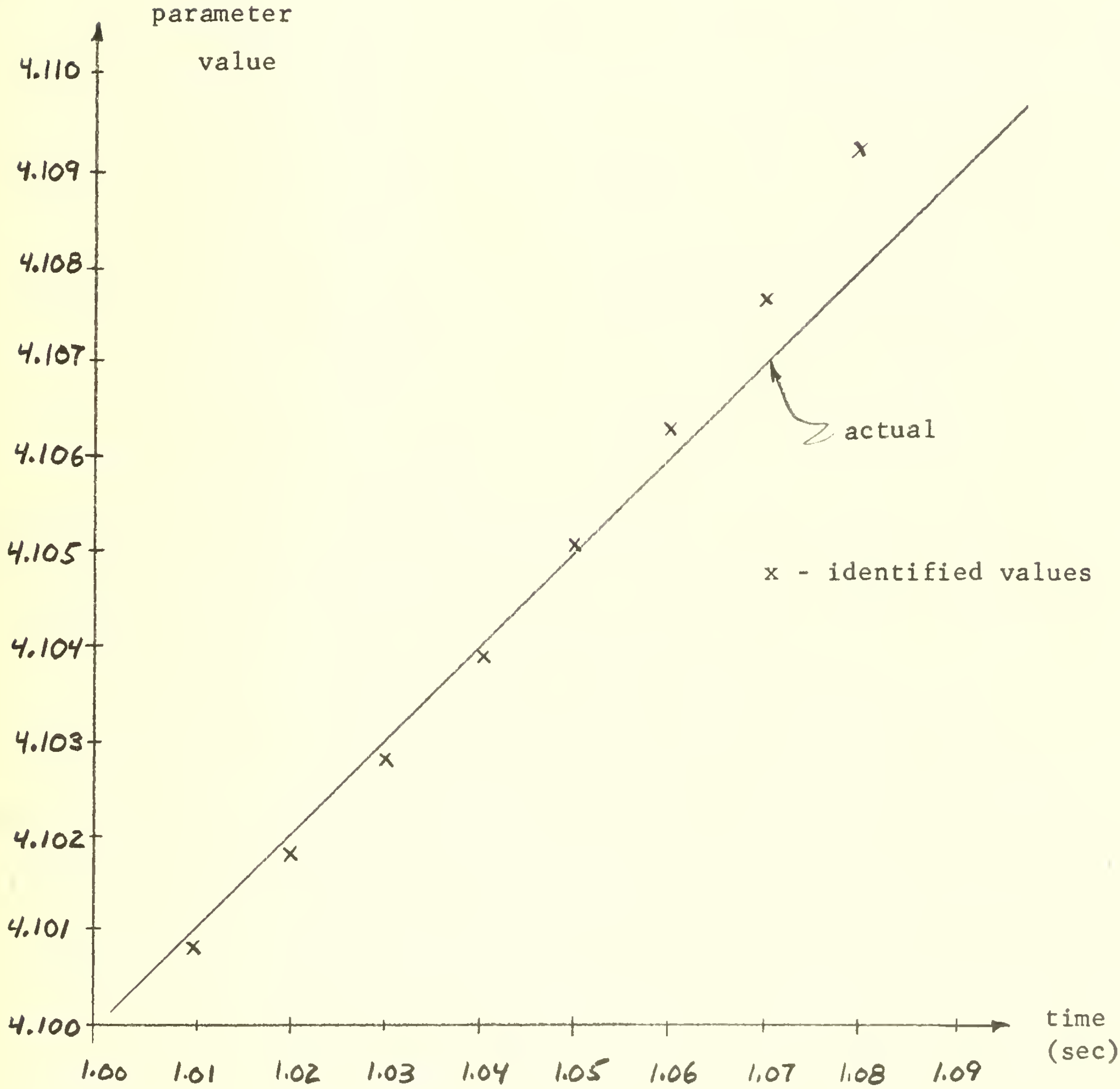


$\alpha_0 = 4.00$

$\alpha = 4.00$ for $t < T$

$\alpha = 4.00 + 0.1t$ for $t \geq T$

(a)



(b)

Fig. (7)

closely for the first five or six samples. After this the accuracy of the identification process begins to fall off. This is in part due to the build up of truncation and round-off errors in the computer simulation. The accuracy of the identification process is diminished as the error between the output of the plant and the output of the model begins to increase. However, it should be noted that when the identification process is incorporated into the adaptive control system, the modification process will be correcting the system after the first few samples. Thus, extreme accuracy in the identification process is only required for the first few sampling intervals after the parameter change occurs and before modification is affected.

The mathematical expression which is used to determine the amount that the parameter varies, equation (22), assumes that;

$$\frac{d[\Delta\alpha(t)]}{dt} \equiv 0$$

This assumption is not valid in this example, thus, some error is introduced into the identification process by assuming that the parameter does not change with time.

Example (4)

In this example, the second order plant of example (3) undergoes a sinusoidal parameter change at time T = 1.0 seconds. Prior to the parameter change $\alpha(t) = 4.0$ and after the parameter is changed

$$\alpha(t) = 4.0 + 0.1 \sin(\omega t)$$

The results of this example are shown in Tables (IVa), (IVb), and (IVc).

Table (IVa)

w = 1.0 radians/second

Time	actual $\alpha(t)$	identified $\alpha_I(t)$	% error in identification
1.01	4.08481	4.084476	0.005066
1.02	4.08526	4.08499	0.005299
1.03	4.08571	4.08554	0.004715
1.04	4.08616	4.08612	0.003197
1.05	4.08677	4.08672	0.000560
1.06	4.08721	4.08809	0.003299
1.07	4.08788	4.08809	0.008982
1.08	4.08829	4.08887	0.016469
1.09	4.088701	4.08973	0.026310
1.10	4.08910	4.09072	0.039111

Table (IVb)

w = 5.0 radians/second

Time	actual $\alpha(t)$	identified $\alpha_I(t)$	% error in identification
1.01	3.90400	3.90403	0.0020275
1.02	3.90545	3.90556	0.0020999
1.03	3.90728	3.90755	0.0034413
1.04	3.90937	3.90997	0.0138803
1.05	3.91170	3.91286	0.0308411
1.06	3.91428	3.91631	0.0563080
1.07	3.91709	3.92041	0.0927920
1.08	3.92014	3.92527	0.1433800
1.09	3.923396	3.93104	0.2120900
1.10	3.92687	3.93793	0.3042000

Table (IVc)

w = 10.0 radians/second

Time	actual $\alpha(t)$	identified $\alpha_I(t)$	% error in identification
1.01	3.9450	3.94555	0.0011119
1.02	3.9357	3.93663	0.0218220
1.03	3.9293	3.92790	0.0537020
1.04	3.9234	3.91943	0.0967230
1.05	3.9181	3.91133	0.1504080
1.06	3.9134	3.90367	0.2136500

These results are plotted on figures (8a), (8b), and (8c) respectively. The solid curves represent the actual parameter values and the crosses represent the parameter values determined by the identification process.

This example has shown that the identification methods introduced in this paper can follow a sinusoidally varying parameter change with a high degree of accuracy. The accuracy of the identification process is better for a slowly varying sinusoid than it is for a sinusoid with a higher frequency.

Examples (3) and (4) have explicitly shown that the parameter identification process can follow a time varying parameter change with a high degree of precision. This high degree of accuracy generally holds for the first five or six sampling intervals unless the parameter variation is rapid. It should be remembered that the modification process requires very accurate information from the identification only in the first few sampling intervals. After this time, some modification has been made to the system so that the identification process must begin all over. Identification in an adaptive control system is discontinuous, whereas the identification employed in these examples was continuous.

$\alpha(t) = 4.0$ for $t < T$

$\alpha(t) = 4.0 + 0.1 \sin(1.0t)$ for $t \geq T$

x - identified parameter values

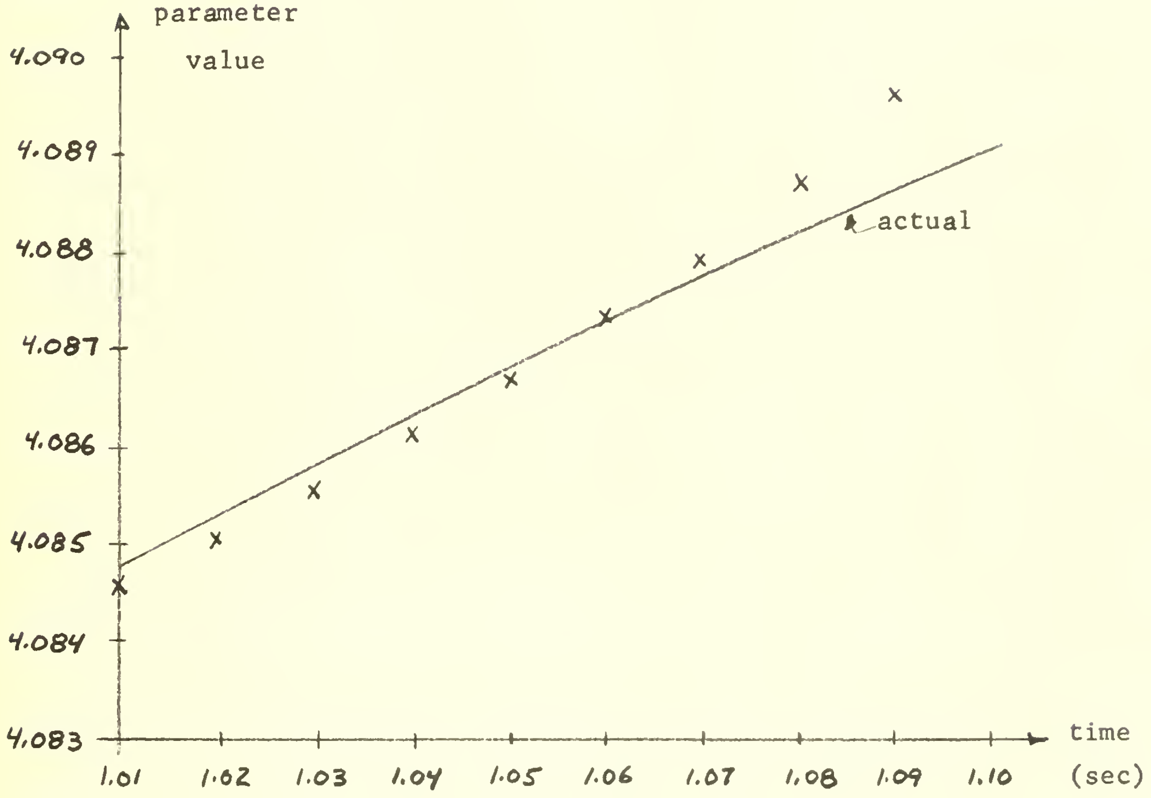


Fig. (8a)

$\alpha(t) = 4.0$ for $t < T$

$\alpha(t) = 4.0 + 0.1 \sin(5.0t)$ for $t \geq T$

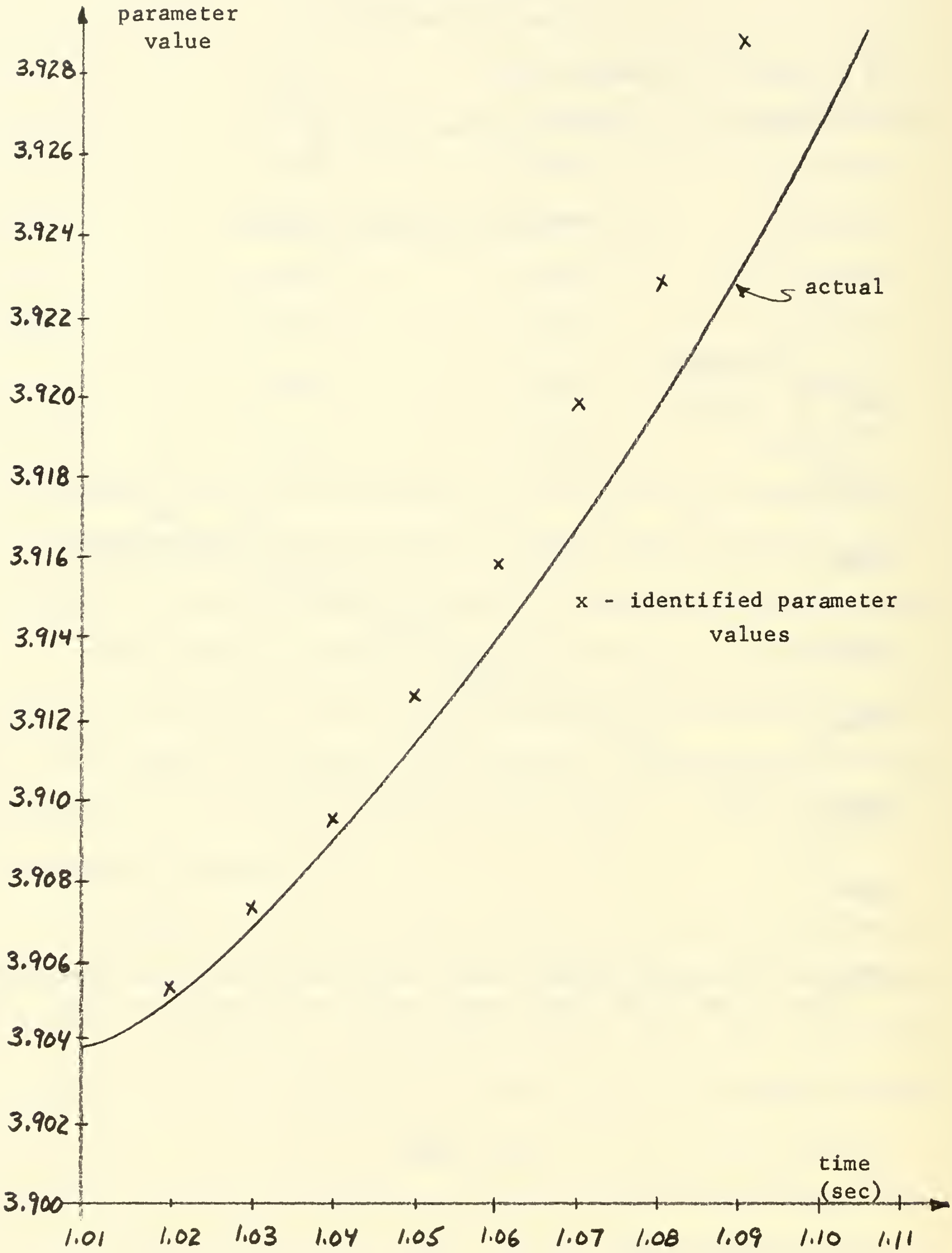


Fig. (8b)

$\alpha(t) = 4.0$ for $t < T$

$\alpha(t) = 4.0 + 0.1 \sin(10.0t)$ for $t \geq T$

x - identified parameter values

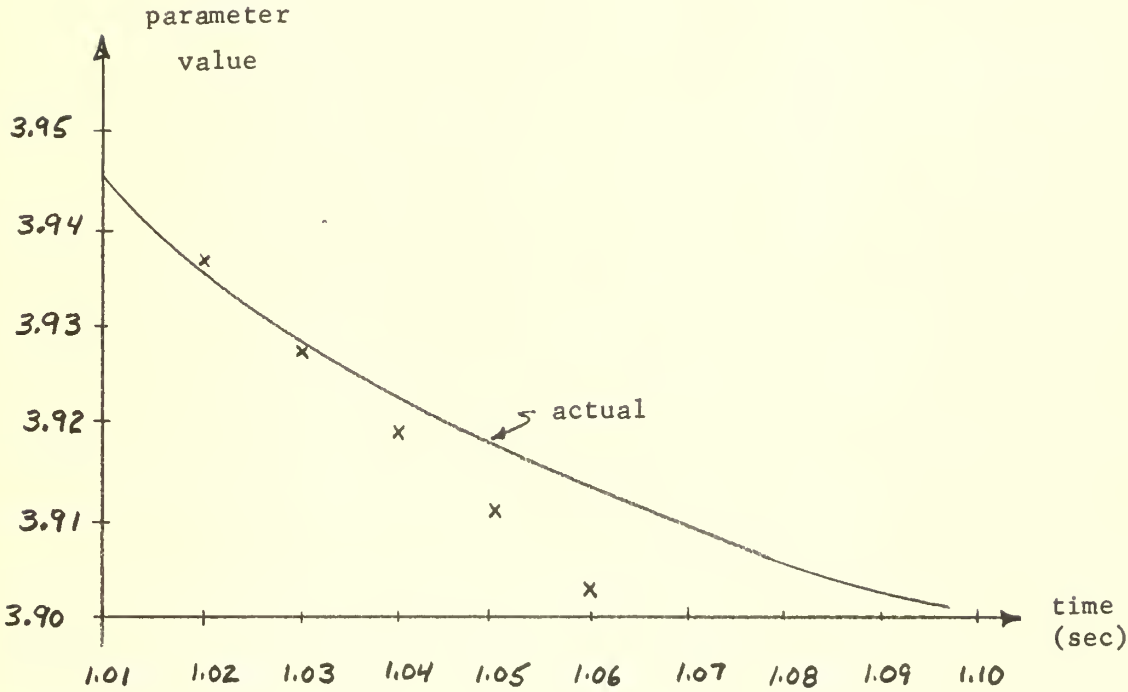


Fig. (8c)

Example (5) First order plant, two variable parameters

This example serves to show the effectiveness of the parameter identification process on the two variable parameter plant of figure (9). The state equations for a digital computer simulation are;

$$\dot{x}_m(t) = -(G_o + A_o)\, x_m(t) + E(t) G_o$$

$$\dot{x}_p(t) = -(G + A)\, x_p(t) + E(t) G$$

$$\dot{V}_1(t) = -(G_o + A_o) V_1(t) - A_o x_m(t)$$

$$\dot{V}_2(t) = -(G_o + A_o) V_2(t) - G_o x_m(t) + G_o E(t)$$

where $A_o = 0.5$, $G_o = 2.0$, $\Delta t = 0.01$ seconds and the reference input is a step input of magnitude ten. At time $T = 1.50$ seconds, A was changed from 0.50 to 0.51. or a 1.96% parameter change, and G was changed from 2.00 to 2.05, or a 2.5% parameter change. From equations (26) and (27) the following results:

$$\begin{bmatrix} \frac{\Delta A}{A_o} \\ \frac{\Delta G}{G_o} \end{bmatrix} \quad \begin{bmatrix} V_1(T+\Delta t) - V_1(T) & V_2(T+\Delta t) - V_2(T) \\ V_1(T+2\Delta t) - V_1(T+\Delta t) & V_2(T+2\Delta t) - V_2(T+\Delta t) \end{bmatrix}^{-1} \begin{bmatrix} e(T+\Delta t) - e(T) \\ e(T+2\Delta t) - e(T+\Delta t) \end{bmatrix}$$

$$C = V^{-1} E$$

$$\Delta A = A_o C(1,1) \qquad \Delta G = G_o C(2,1)$$

These equations form the mathematical expressions used in the identification process.

The results of this example are shown in table (V). The accuray of the identification is given by the percent error in the identification process which was defined previously.

Two Variable Parameter Identification

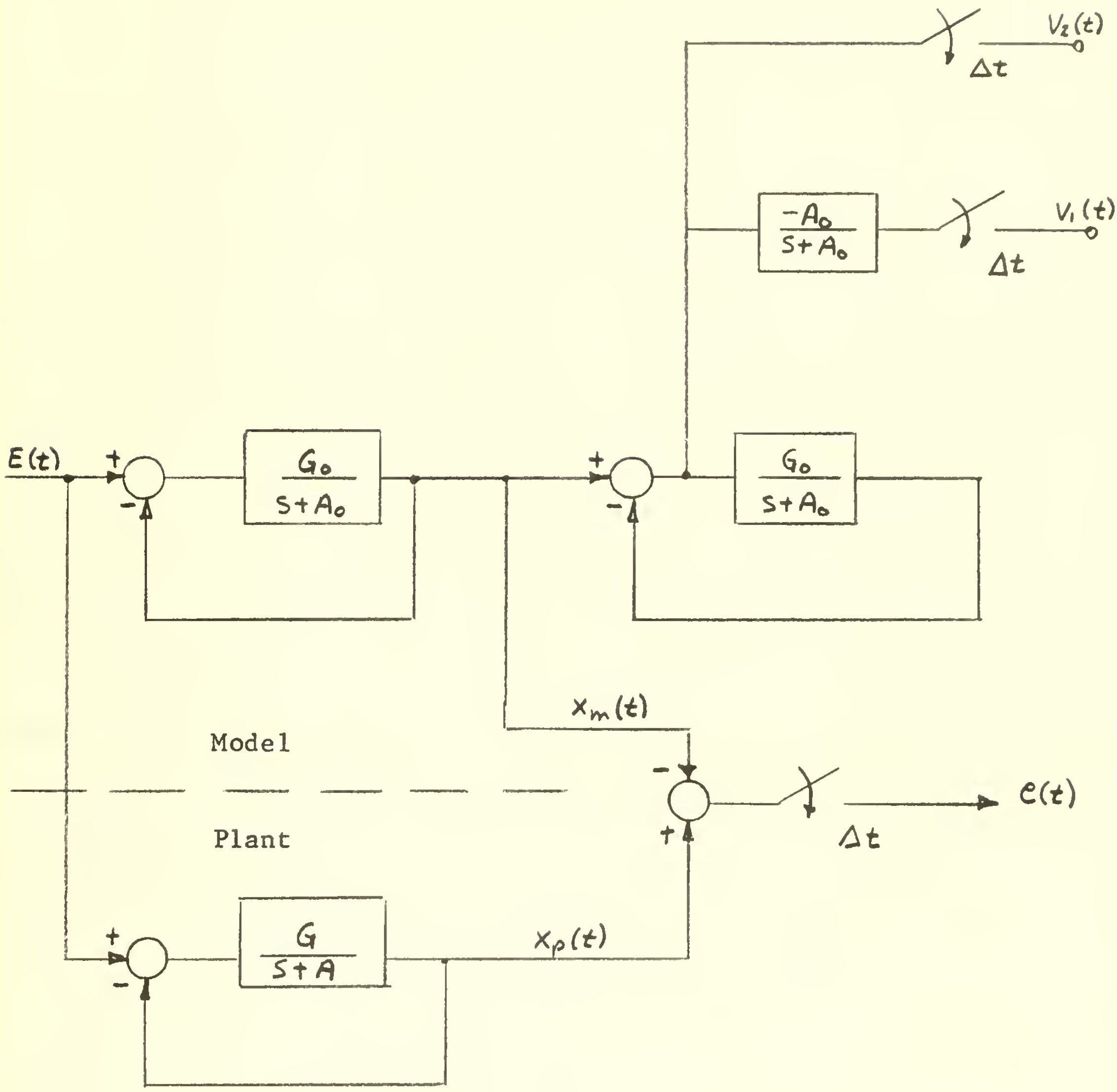


Fig. (9)

Table (V)

Time	identified A	% error in identification of A	identified G	% error in identification of G
1.52	0.51200	0.39218	2.0572	0.35123
1.54	0.50301	0.60130	2.0097	0.48609
1.56	0.50301	0.60130	2.0098	0.48938
1.58	0.49767	0.66536	0.9904	0.48960
1.60	0.49757	0.68635	1.9899	0.50435

It should be noted that the identification for a two variable parameter plant occurs on every other sampling interval. This is because two samples are required to perform the identification as there are two simultaneous equations which must be solved in order to perform the identification.

This example has shown that the identification of two variable parameters is not as accurate as that for only one variable parameter. Much of the error in the two variable parameter case arises from the matrix techniques which are used in the computer simulation. However, the accuracy of the identification is sufficient for the adaptive control systems described in this paper. In the adaptive control system, modification takes place as soon as the parameter is identified, so that only the first identified values are of importance.

6. Adaptive Control Systems

The previous sections have shown that the adaptive control system must have a plant and sensitivity model. The adaptive system must have an identifier which receives information from the error signal and the sensitivity model. The adaptive system must also have a means of triggering the sensitivity model when the error signal ceases to be zero. The amount of success of the adaptive control system to minimize the error is in direct proportion to the accuracy of the identification process. The accuracy of the identification depends upon the sampling interval which is used and upon the accuracy of the computations used in the digital simulation of the models and the identifier.

In this paper, two adaptive techniques are proposed. Both of these techniques are identical in the methods used to decide whether or not a parameter has changed and the methods used in the identification of the parameter change. The difference in these two techniques lies in the method used to modify the system to compensate for the parameter change.

6.1 Adaptive Technique One.

The adaptive control system which employs modification technique one is shown in figure (10). In this method of modification, the adaptive controller uses the information which it receives from the identifier to physically change one of the parameters of the plant. One way to accomplish this is to readjust the parameter which has varied. The amount of readjustment is determined by the identification process. The identification and modification processes continue until the error is reduced to zero or a suitable value. It should be noted that the plant and the model both generate signals continuously while modification and identification are in process.

Adaptive Technique No. 1

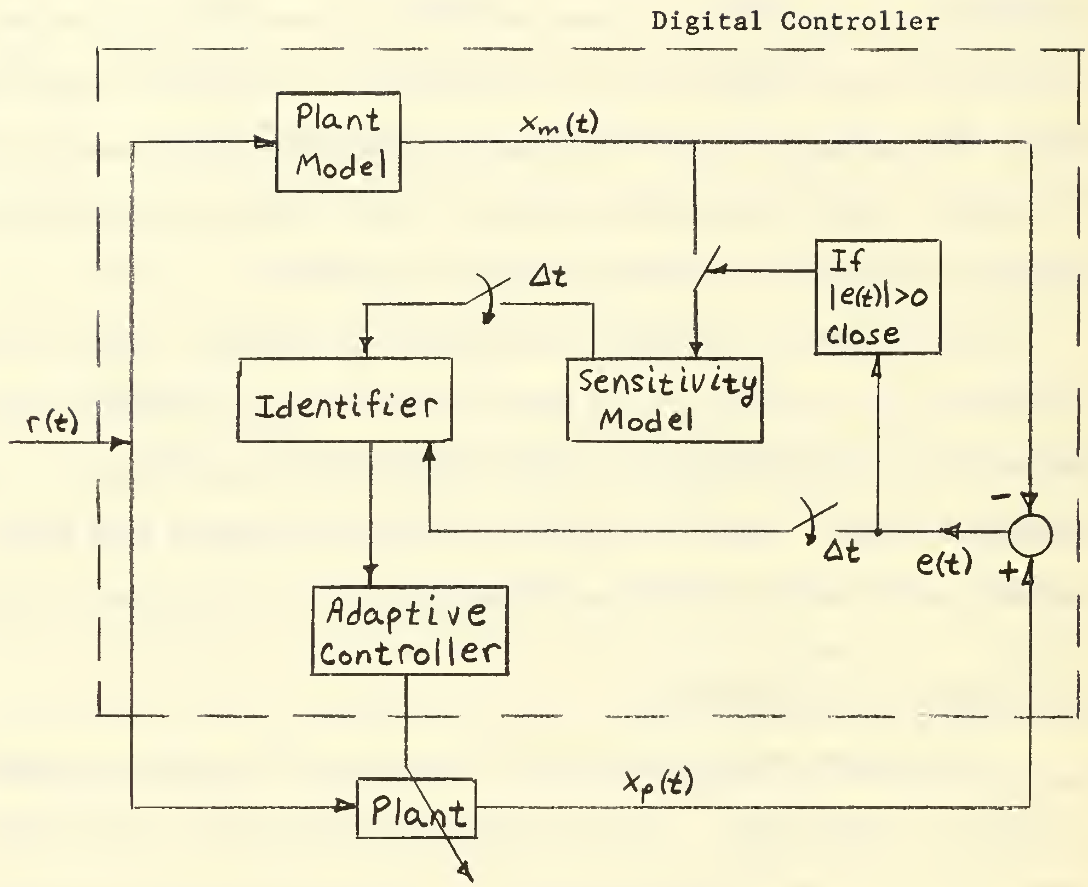


Fig. (10)

If the plant which is to be employed in the adaptive scheme has one parameter which is variable and a different parameter which is adjustable, then parameter plane theory may be used in the modification process. When the plant consists of a dominant pair of complex poles which must be maintained invarient as a parameter of the plant varies, and the characteristic equation of the plant exhibits singular lines on the parameter plane, then adjustment may be made based on the singular line for the dominant complex poles in question. When the plant possesses an invarient real root which is dominant, then adjustment may be made based on a constant sigma line of the parameter plane. Singular lines and constant sigma lines plot as straight lines on the parameter plane. Thus, they provide a linear straight line relationship for parameter adjustment.

Example (1) First order plant, one variable parameter.

This example serves to illustrate the use of adaptive technique one on the low pass filter of example (1) of the previous section. The adaptive control system was simulated on an IBM-360 digital computer using the Fortran flow graph shown in figure (11).

For this example, the variable parameter α undergoes a step change at time T = 1.5 seconds from $\alpha = 0.2$ to $\alpha = 0.19$. The amount of the parameter change which is determined by the identification process is given by;

$$\Delta\alpha_I = \alpha_0 \frac{e(T+\Delta t) - e(T)}{V(T+\Delta t) - V(T)}$$

where $\Delta t = 0.10$ seconds. The modification process, which is accomplished by the adaptive controller, readjusts the parameter in the plant by the amount $\Delta\alpha_I$ determined by identification.

Figures (12a), (12b), and (12c) show the error between the plant output and the model output for a step, ramp, and sinusoidal reference input

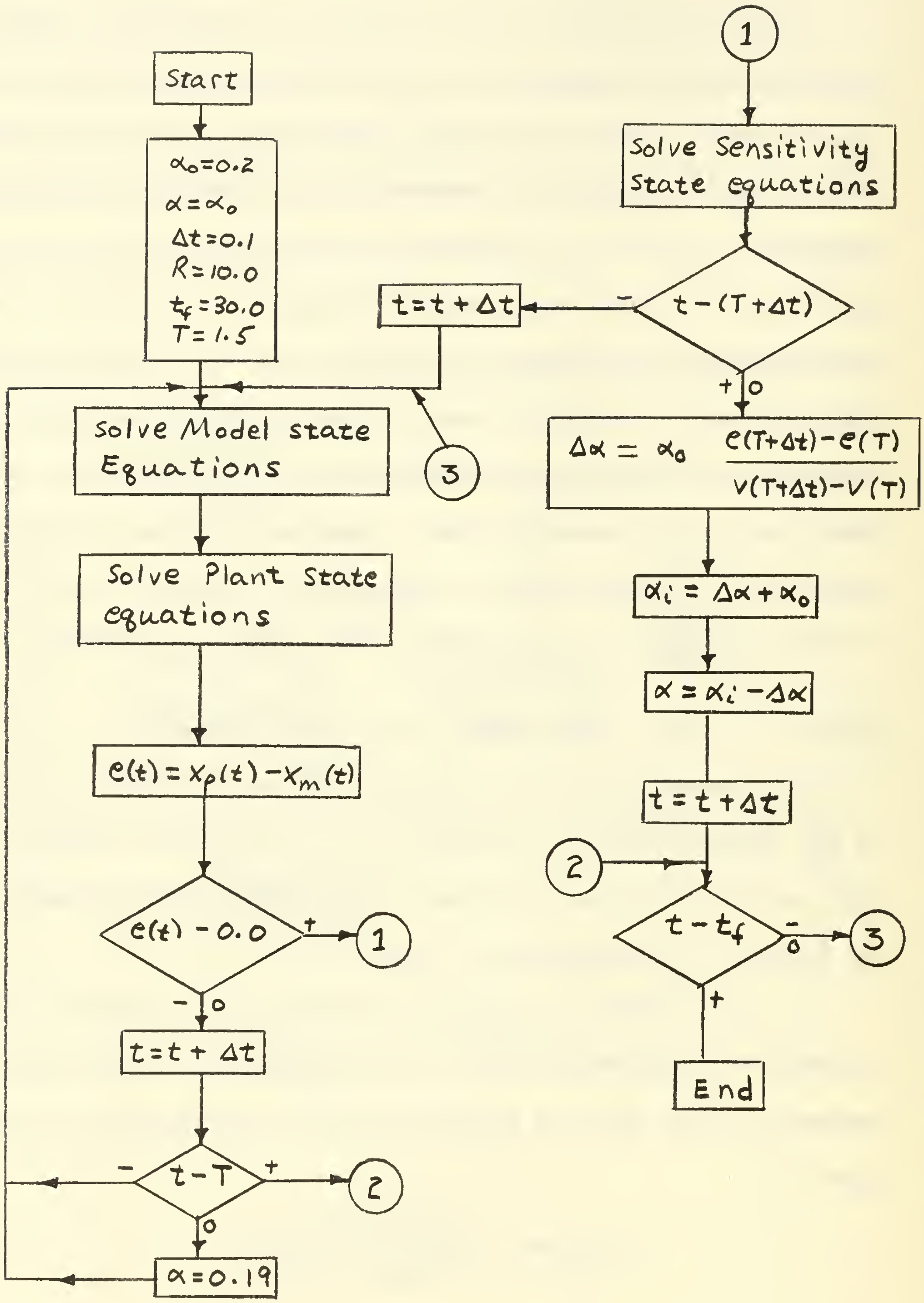
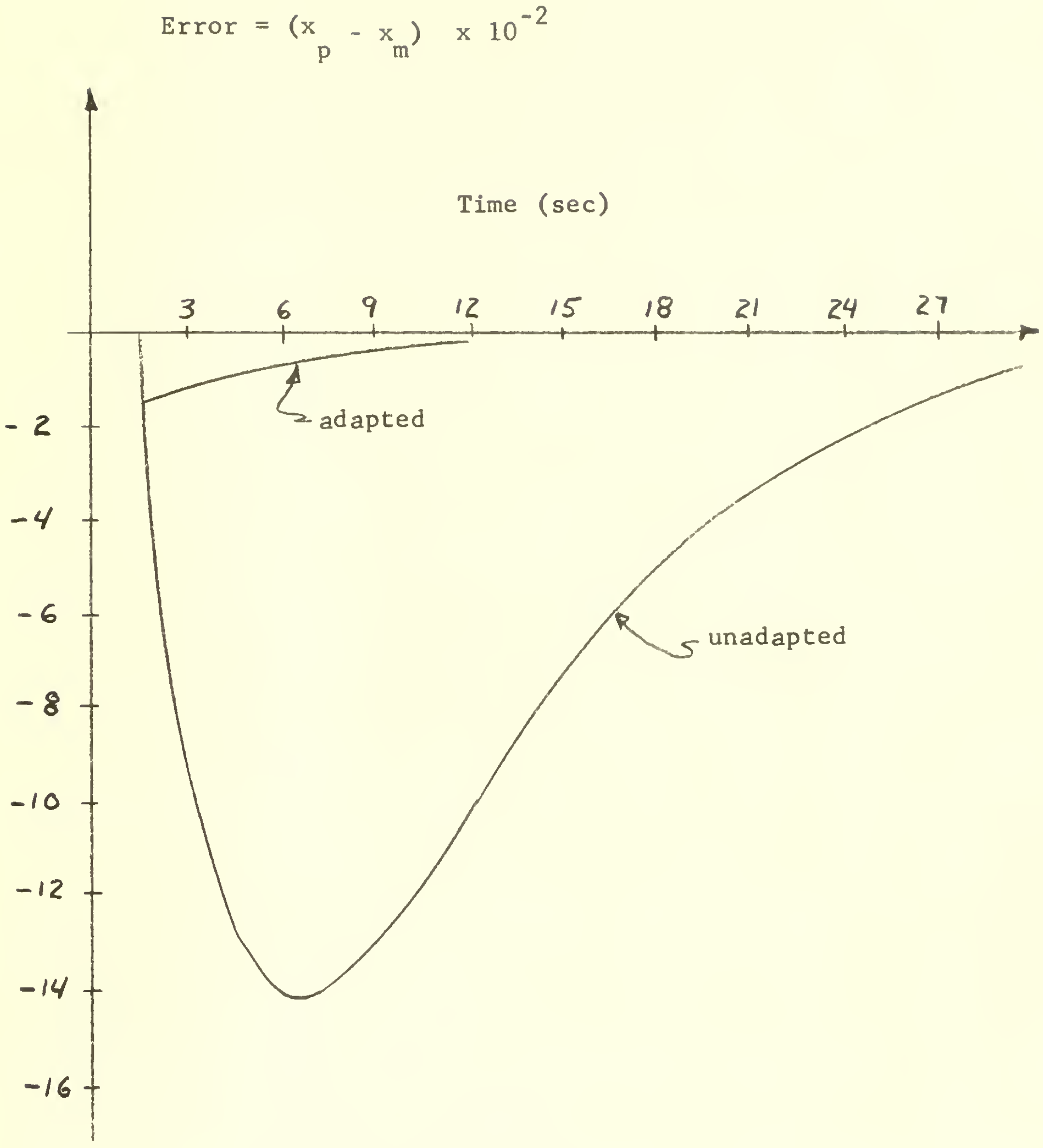
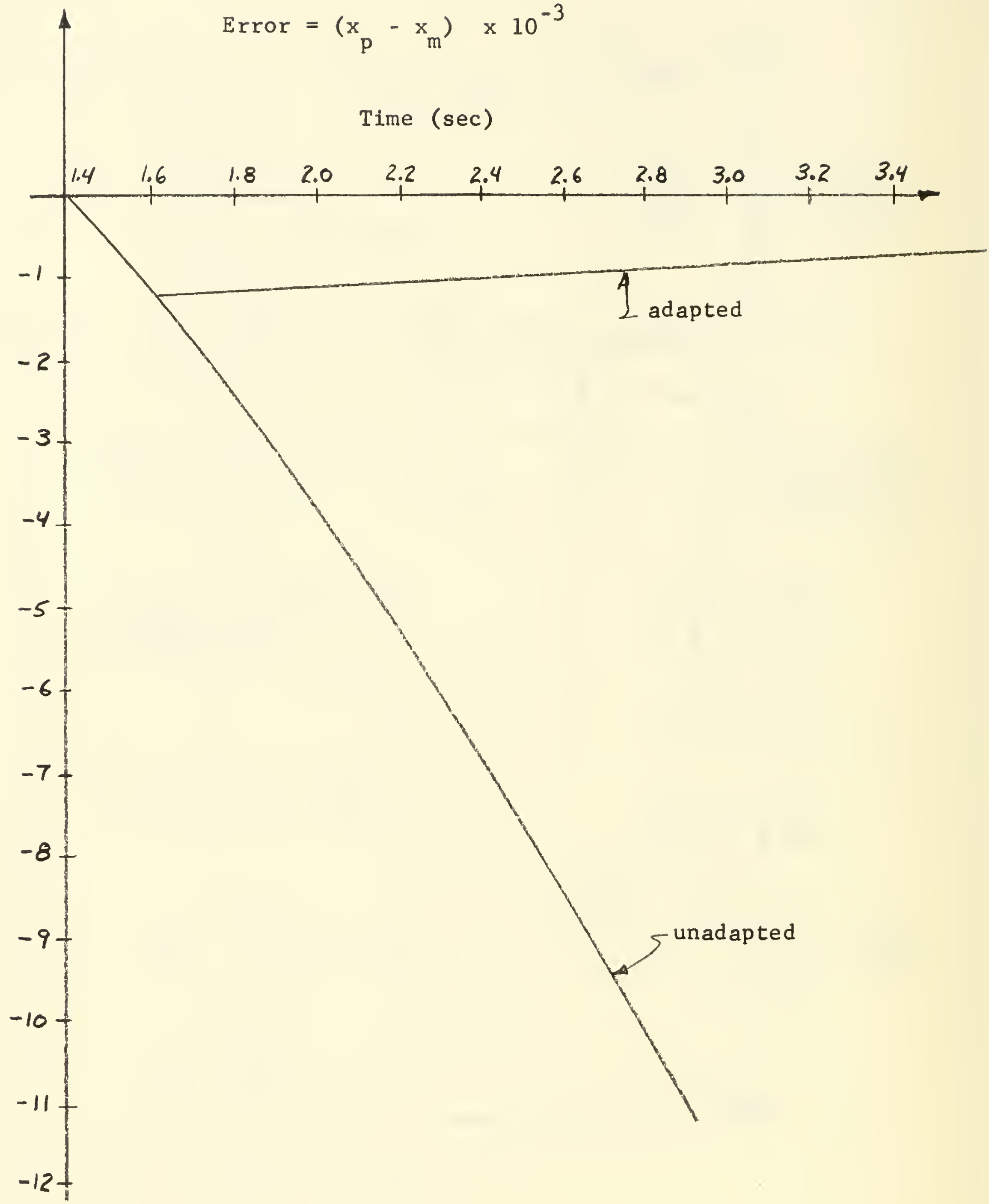

Start
α₀=0.2
α=α₀
Δt=0.1
R=10.0
t_f=30.0
T=1.5
Solve Model state Equations
Solve Plant state equations
e(t)=x_p(t)−x_m(t)
e(t) − 0.0
t=t+Δt
t−T
α=0.19
1
Solve Sensitivity State equations
t−(T+Δt)
t=t+Δt
Δα = α₀ (e(T+Δt)−e(T))/(V(T+Δt)−V(T))
α_i = Δα+α₀
α = α_i −Δα
t=t+Δt
t−t_f
End
2
3

Fig. (11)



r(t) = 10.0 for all time

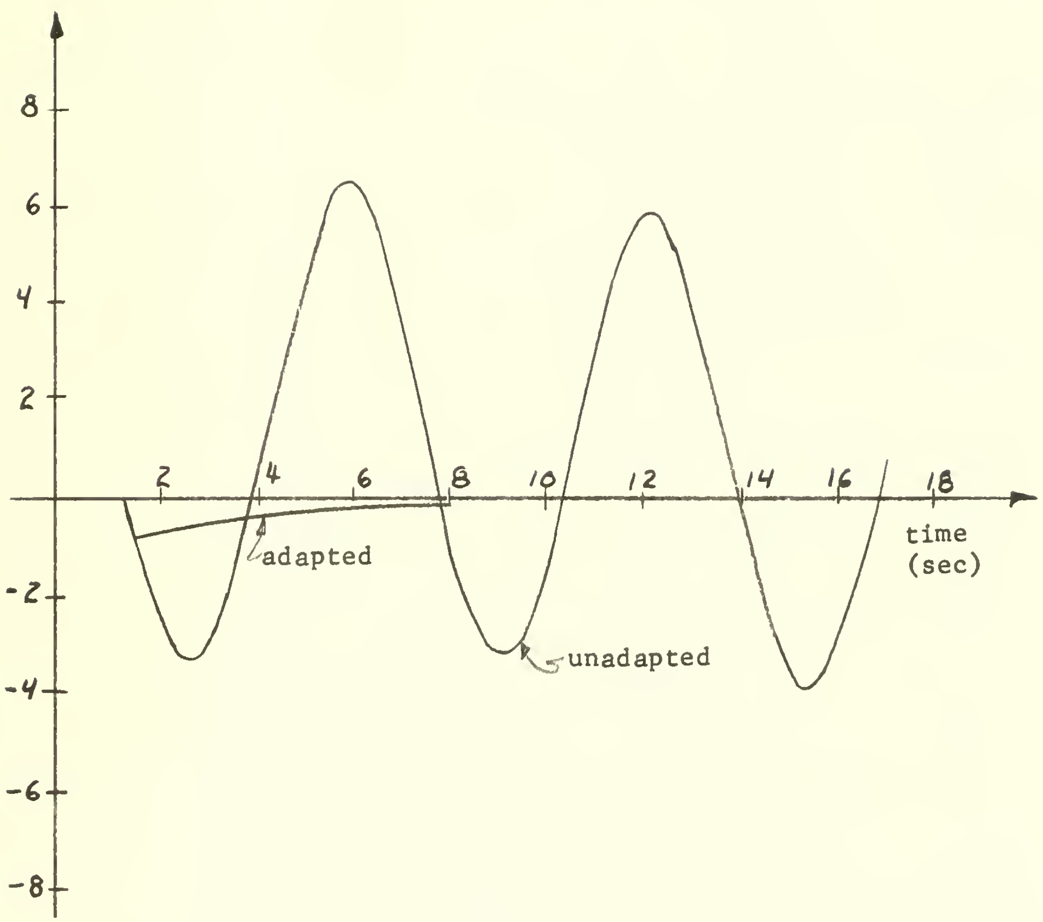
Fig. (12a)



$r(t) = 0.5t$

Fig. (12b)

Error = $(x_p - x_m) \times 10^{-2}$



r(t) = 5.0sin(t)

Fig. (12c)

respectively. The error is shown for the adapted and the unadapted systems. In the unadapted system, the parameter changes at the time given above, but, modification of the system is not accomplished. Examination of these figures shows that the adaptive scheme produces a marked improvement on the performance of the plant when one of the plant parameters undergoes a change.

Example (2) First order plant, two variable parameters

For this example, the system shown in figure (9) is adapted by technique one. At time $T = 1.5$ seconds, the parameter A was changed from 0.5 to 0.51 and the parameter G was changed from 2.00 to 2.05. Both of the parameters were changed simultaneously. As in example (1), both of the parameters were readjusted by the amount determined by the identification process.

The results of this example are shown in figure (13). The error between the plant and the model output for the adapted and the unadapted systems are shown. As in the previous example, the adaptive scheme produces a significant improvement in the plant. That is, the error is reduced to zero very rapidly and the plant follows the model very accurately.

Example (3) Third order plant, one variable parameter

The third order plant of figure (14a) has one parameter which is variable, α , and a different parameter, β , which is adjustable. The design specifications on this system require the plant to maintain a dominant pair of complex roots with a ζ of 0.5 and w_n of 4.0. Due to changes in the environmental conditions, the parameter will vary about its nominal value of $\alpha_o = 5.10$. As the location of the dominant roots must be maintained invarient, the system will be used in a self-adaptive mode with variations in α being compensated for by adjusting the value of β .

$$\text{Error} = (x_p - x_m) \quad \times 10^{-3}$$

$$r(t) = 10.0\ u(t)$$

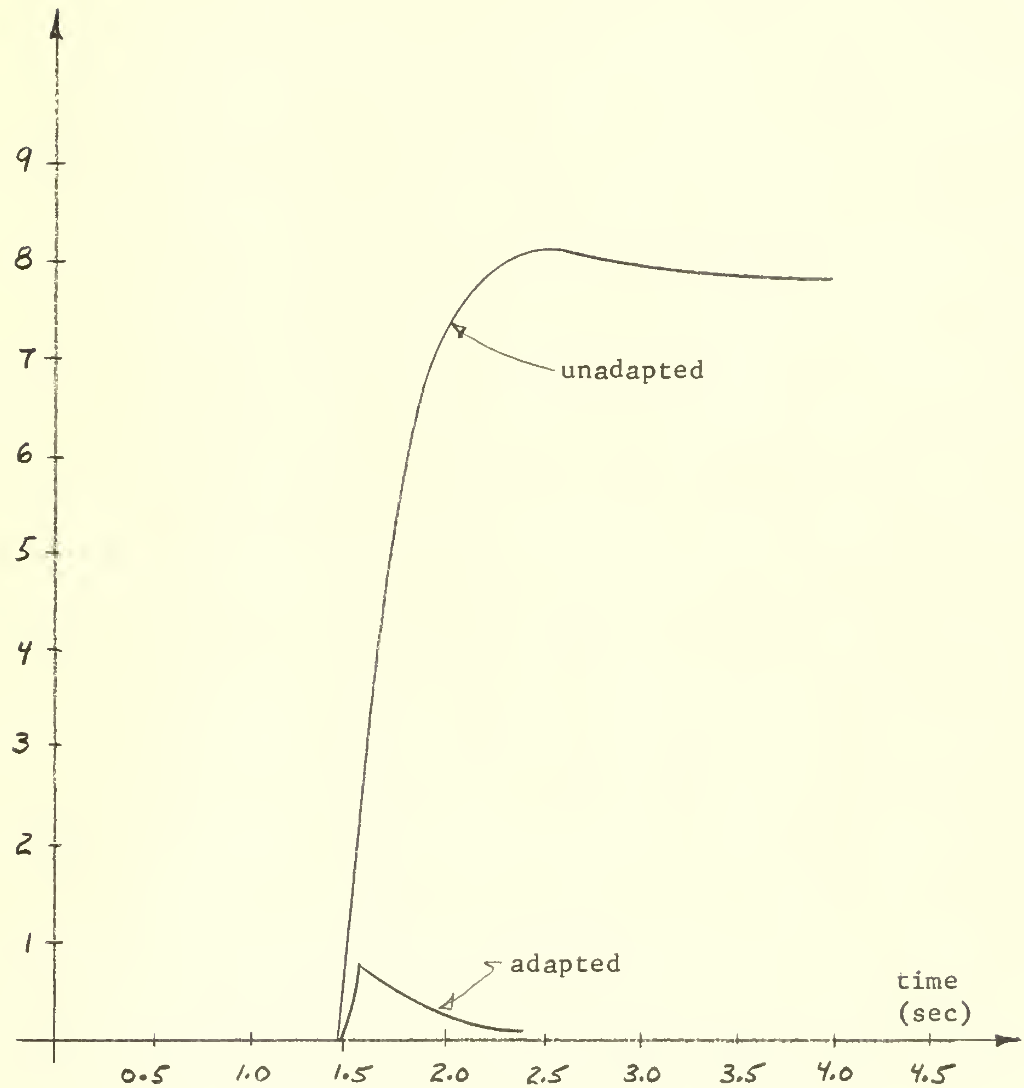


Fig. (13)

Bodie, [11], has shown the criteria for determining whether or not a plant characteristic equation exhibits singular lines on the parameter plane. A singular line plots as a straight line and represents a constant ζ constant ω_n line. That is, there are an infinite number of α, β pairs which lie on the singular line and will give exactly the same pair of dominant complex roots.

The plant of figure (14a) possesses a singular line for $\zeta = 0.50$ and $w_n = 4.00$. The equation of the singular line is given by;

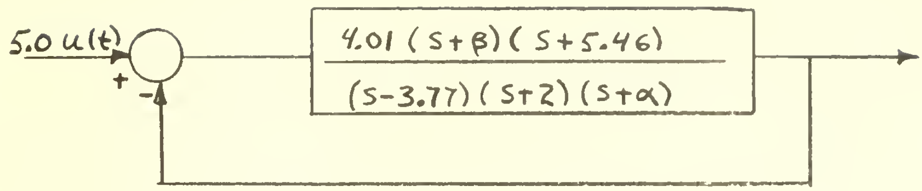
$$\beta = 1.156 + 0.935\,\alpha$$

At time T = 0.1 seconds, the parameter α undergoes a change from 5.10 to 4.50, or a 12% parameter change, which is identified by the identification process as 4.50043. The adaptive controller then uses the value of α determined by the identification in the equation of the singular line to determine the value that β must be adjusted to in order to maintain the dominant complex roots desired.

Figure (14b) shows the error between the plant and the model outputs for the adapted and the unadapted systems. In the unadapted system, parameter α underwent a change, but β was not adjusted to compensate for the variation in α. Singular line theory provides a relatively simple means of performing the modification process. Of course not all systems possess singular lines, and the methods used to determine if singular lines exist are rather laborious.

Example (4) Third order system, one variable parameter

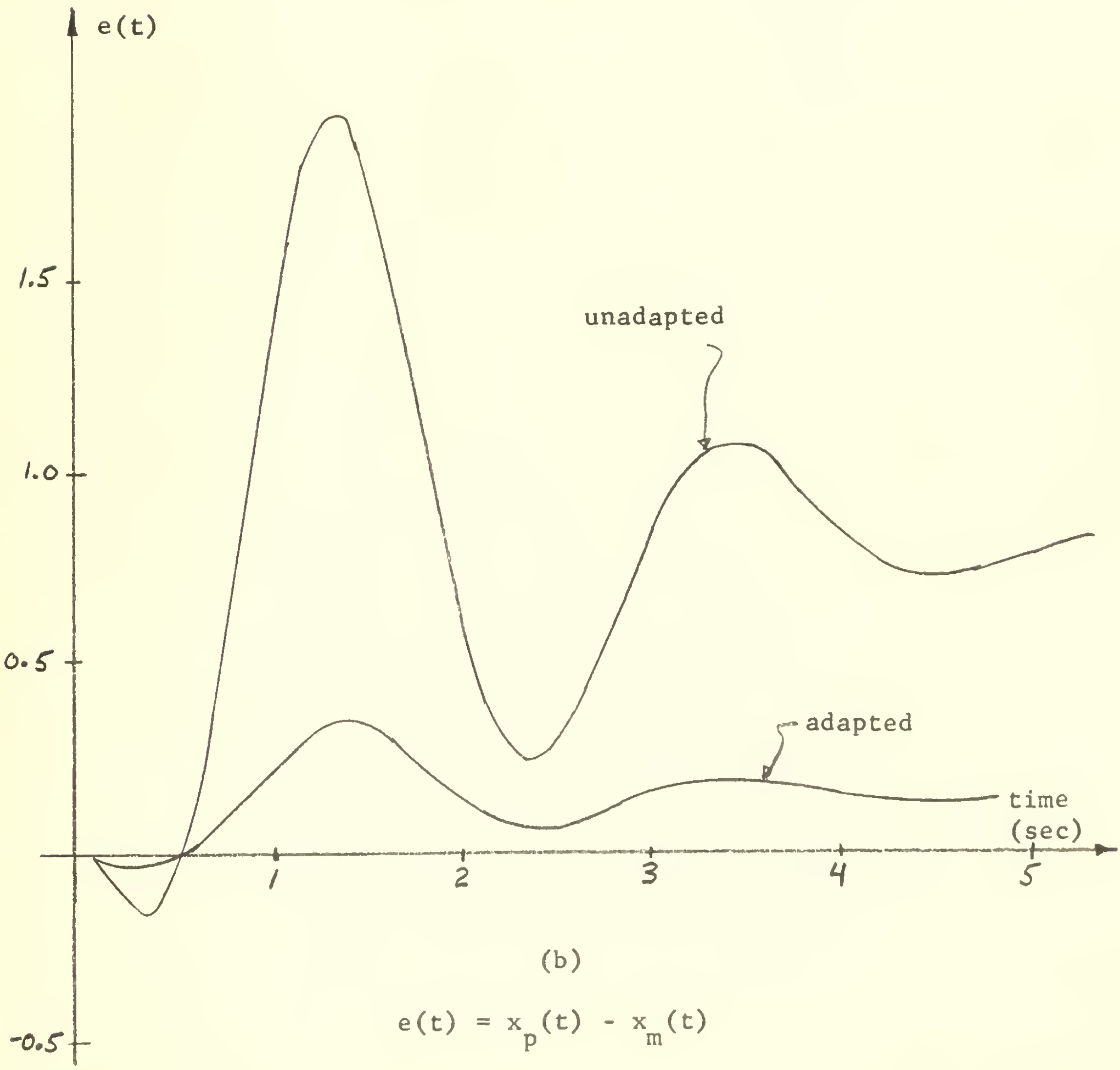
Many control system designs require that a real root be the dominant root. The plant of figure (15a) is required to have a dominant, invarient, real root at sigma equals 2.0 irregardless of how any of the other roots may vary. In this plant, α, will vary with changing environmental



(a)

$\alpha_o = 5.10$

$\beta_o = 5.93$



(b)

$e(t) = x_p(t) - x_m(t)$

Fig. (14)

conditions, and β is an adjustable parameter. The equation of the constant sigma equal 2.0 line on the parameter plane is given by;

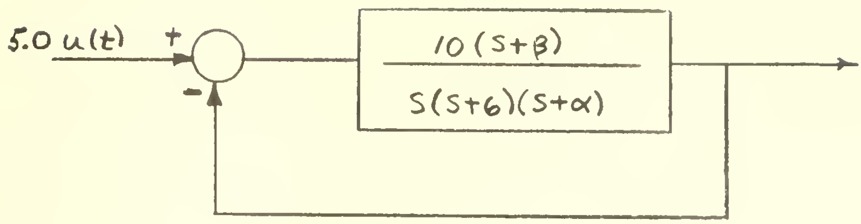
$$\beta = 0.40 + 0.80\alpha$$

At time T = 0.1 seconds, parameter α undergoes a change from its nominal value of $\alpha_0 = 4.0$ to $\alpha = 3.5$. The identification process determined the new value of α to be 3.49597. Using the new value of α determined by identification, the adaptive controller finds the value of β which is necessary to compensate for the change in α. This new value of β is determined from the equation of the constant sigma line given above.

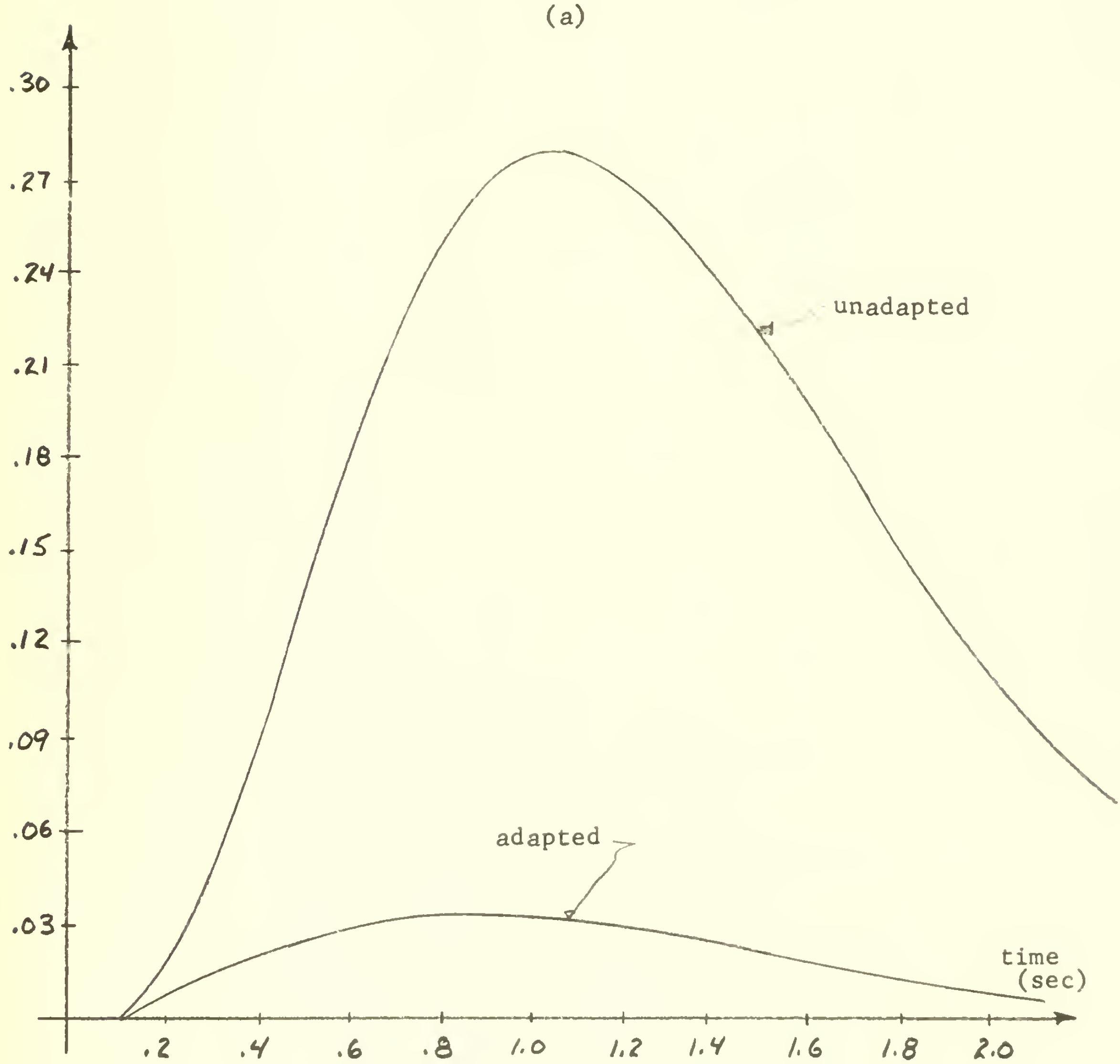
Figure (15b) shows the error between the output of the model and the output of the plant for the adapted and the unadapted systems. The use of constant sigma lines in the modification process has proved to be very effective in reducing the error caused by a parameter change in the plant. This method is only effective when the system response has a zero steady state error as a change in the parameters will change the steady state error in other systems and the plant will be unable to follow the model.

In general, there is a great improvement in the performance of the plant when adaptive technique one is implemented for the modification process. In all of the previous examples the error between the output of the plant and the output of the model was decreased to almost zero in a very short period of time. It should be noted that the implementation of this adaptive technique requires at least one parameter which can be physically adjusted. If the adjustable parameter is not the parameter which is subject to variation, then modification must be made based upon either a singular line or a constant sigma line. If the plant does not possess singular lines on the parameter plane, and a pair of dominant complex roots must be maintained invarient, consideration should be given to re-designing

$$\text{Error} = x_p - x_m$$



(a)



(b)

Fig. (15)

the plant so that it does possess singular lines, or the use of the technique discussed in the next section.

6.2 Adaptive Technique Two

In adaptive technique two, shown in figure (16), the adaptive controller uses the information provided by the identification process to generate an optimum control, u*(t), which when put into the plant, will compensate for the parameter variation. This optimum control is for the time interval $T \leq t \leq t_f$, where t_f is the time at which the plant operation ends or the time at which a second parameter change occurs. Note that only one control history need be generated if only one parameter change occurs.

The optimum control can be generated by many means. As the overall controller, or the self-adaptive portion of the system, is digital in nature, a numerical technique such as the method of steepest descent can be implemented very easily. In the use of a numerical technique, the adaptive controller uses the information from the identification process to generate an error history and then minimize the performance index

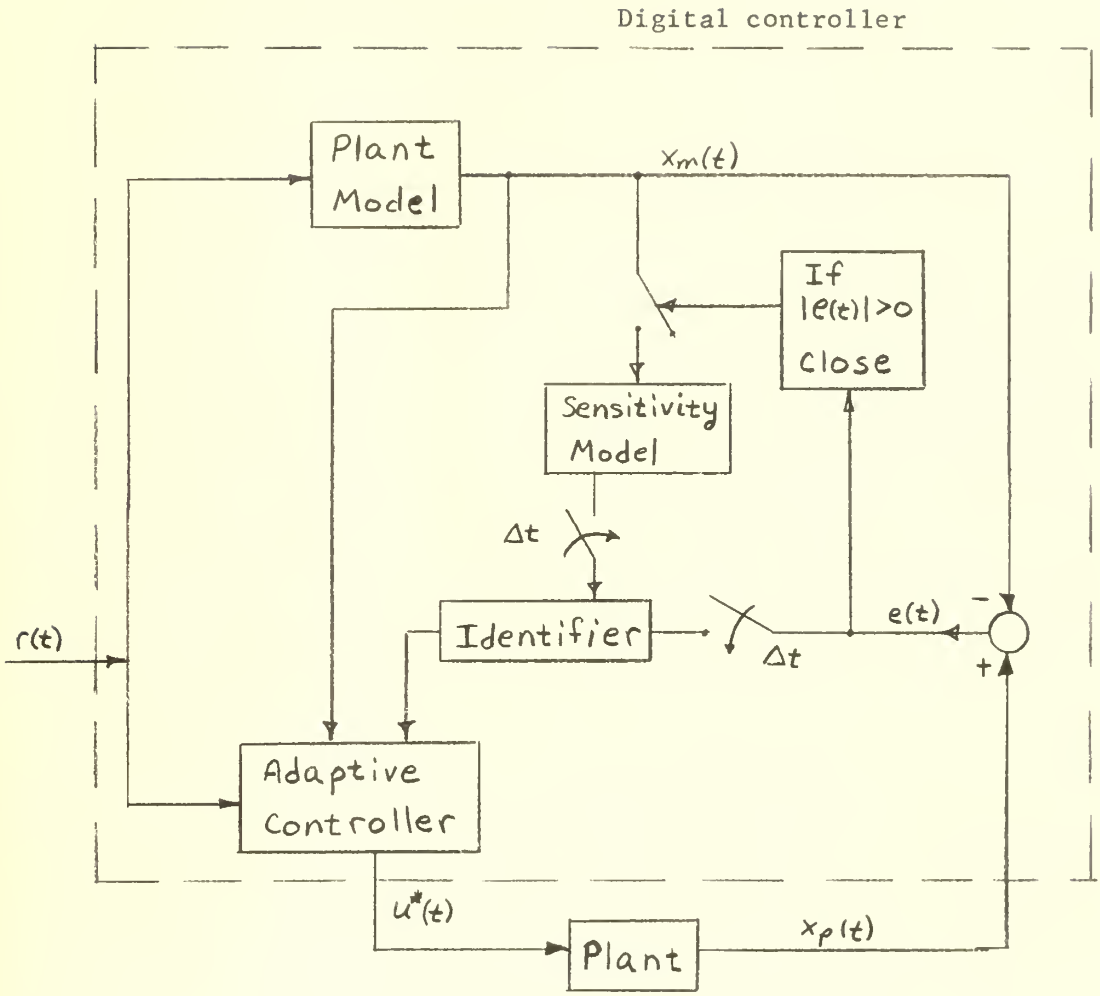
$$J = \frac{1}{2} \int_T^{t_f} e^2(t)\, dt \qquad (33)$$

with respect to u(t) to obtain the optimum control. The optimum control which is generated is in open loop form, thus the exact plant state values do not need to be determined.

The control, u(t), for adaptive technique two can also be found by considering the transfer functions of the plant and the model in their Laplace form. Consider a low pass filter with one variable parameter. Let:

G(s) = Transfer function of the plant

G'(s) = Transfer function of the model



Adaptive Technique No. 2

Fig. (16)

$R(s)$ = Input to the model

$U(s)$ = Input to the plant

$X_p(s)$ = Output of the plant

$X_m(s)$ = Output of the model

$G(s) = X_p(s) / U(s)$ and $G'(s) = X_m(s) / R(s)$

The error is given by;

(34) $$e(s) = X_p(s) - X_m(s)$$

Now: $X_p(s) = U(s)G(s)$

$X_m(s) = R(s)G'(s)$

therefore,

(35) $$e(s) = U(s)G(s) - R(s)G'(s)$$

From equation (5), the error can be written as;

$$e(s) = \frac{\Delta\alpha}{\alpha_0} V(s)$$

where $V(s)$ is the sensitivity of the output with respect to parameter α .

Therefore, equation (34) can be written as;

(36) $$\frac{\Delta\alpha}{\alpha_0} V(s) = U(s)G(s) - R(s)G'(s)$$

Equation (36) is solved for $U(s)$ to give;

(37) $$U(s) = R(s)\frac{G'(s)}{G(s)} + \frac{\Delta\alpha}{\alpha_0}\frac{V(s)}{G(s)}$$

From figure (17) it is apparent that,

(38) $$G'(s) = \frac{\alpha_0}{s+\alpha_0} \qquad V(s) = \frac{\alpha_0}{(s+\alpha_0)^2}$$
$$G(s) = \frac{\alpha}{s+\alpha} \qquad R(s) = \frac{R}{s}$$

Substituting equation (38) into equation (37) yields;

(39) $$U(s) = \left(\frac{R\alpha_0}{\alpha}\right)\frac{(s+\alpha)}{s(s+\alpha_0)} + \left(\frac{\Delta\alpha}{\alpha}\right)\frac{(s+\alpha)}{(s+\alpha_0)^2}$$

Time Relationship Derivation of Control Signal

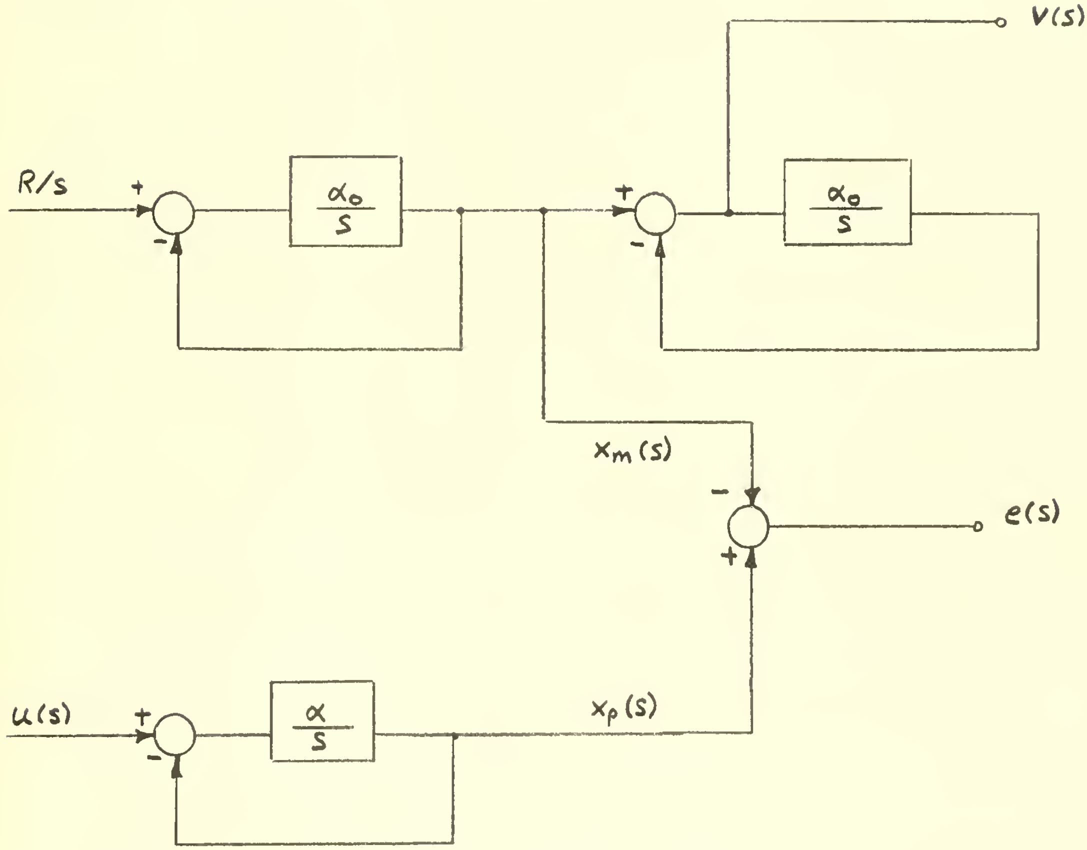
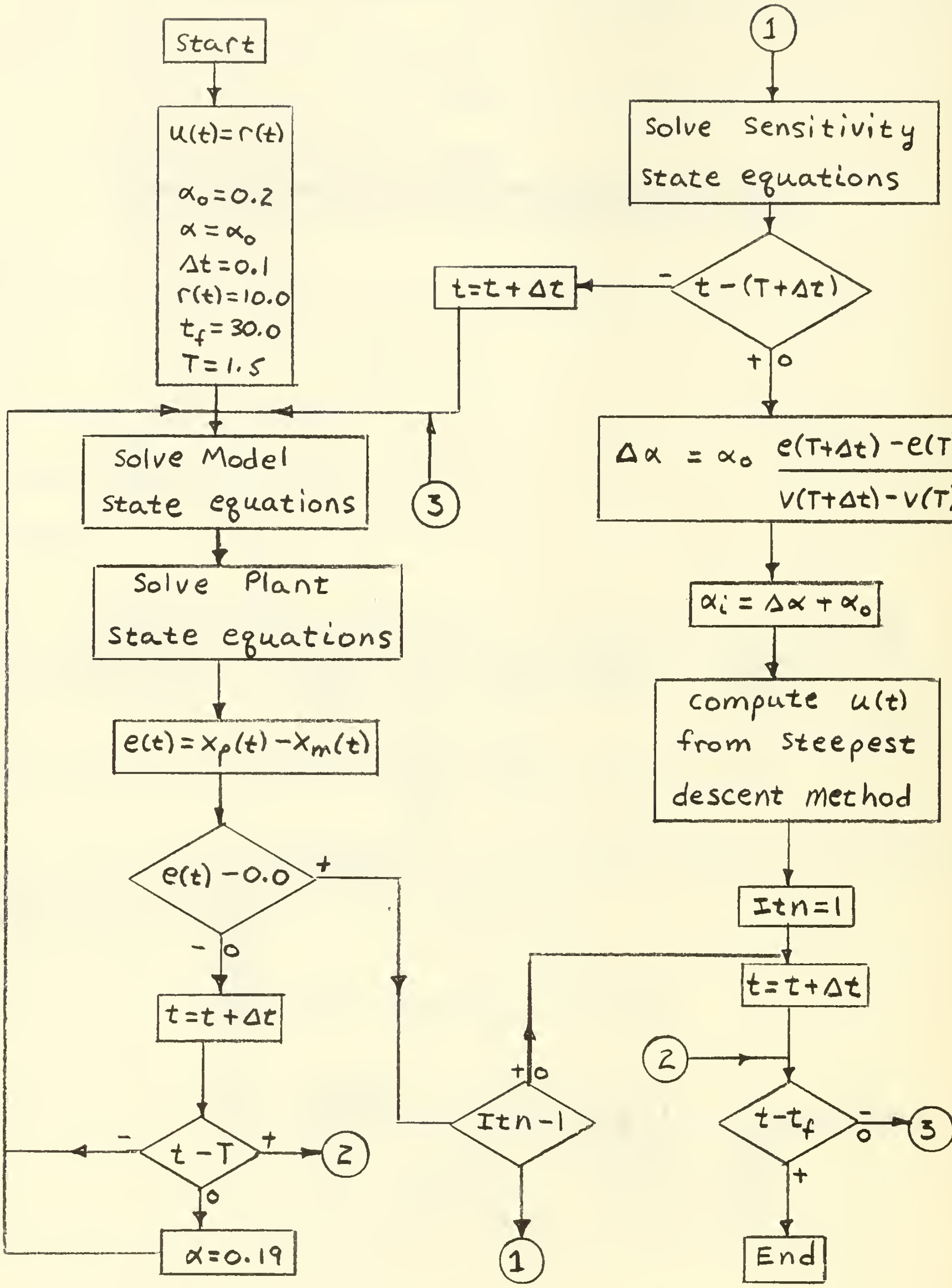


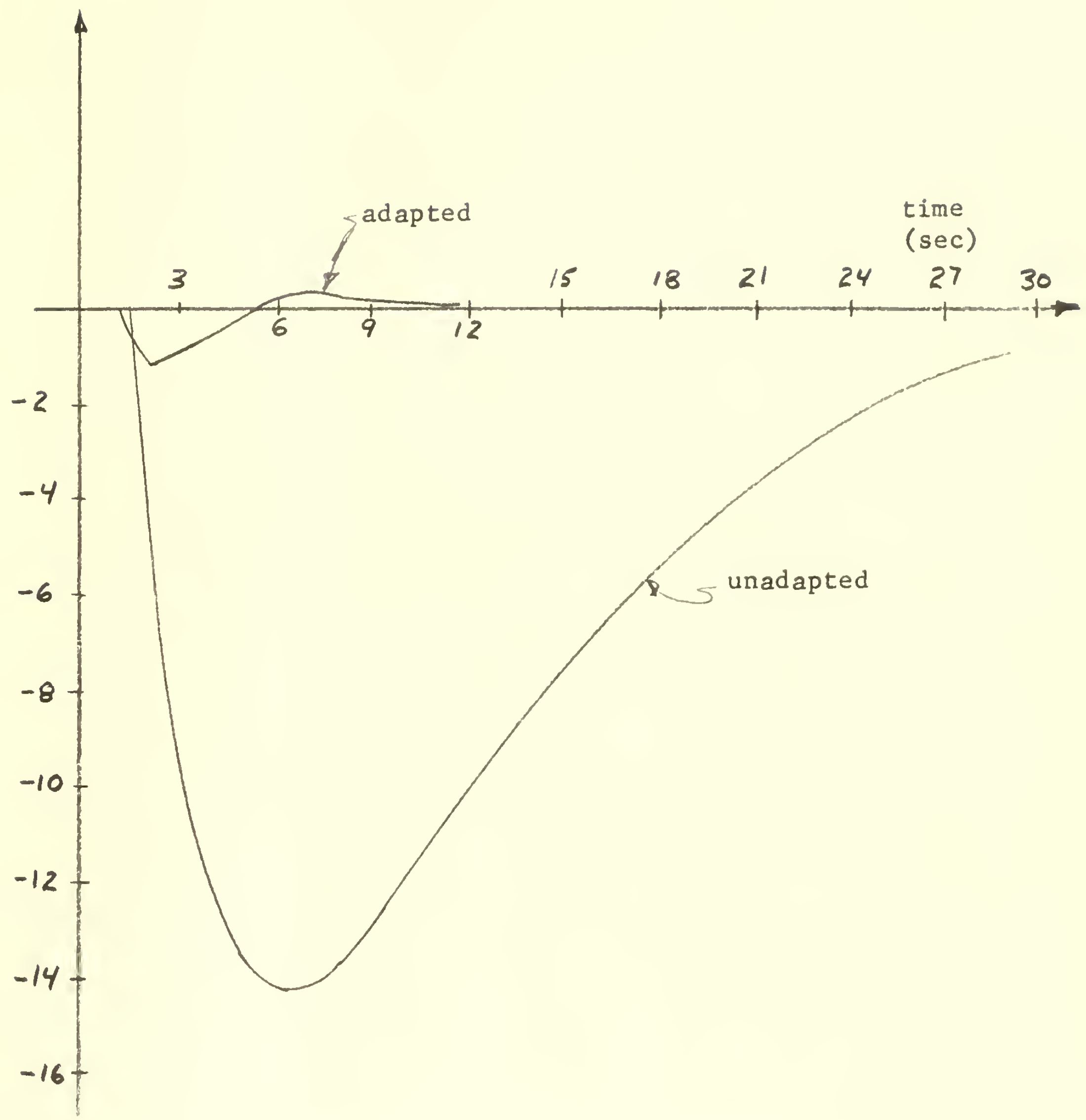
Fig. (17)



Start
u(t)=r(t)
α₀=0.2
α=α₀
Δt=0.1
r(t)=10.0
t_f=30.0
T=1.5
Solve Model state equations
Solve Plant state equations
e(t)=x_p(t)−x_m(t)
e(t)−0.0
t=t+Δt
t−T
α=0.19
Itn−1
Solve Sensitivity state equations
t−(T+Δt)
t=t+Δt
Δα = α₀ (e(T+Δt)−e(T))/(V(T+Δt)−V(T))
α_i = Δα + α₀
compute u(t) from steepest descent method
Itn=1
t=t+Δt
t−t_f
End

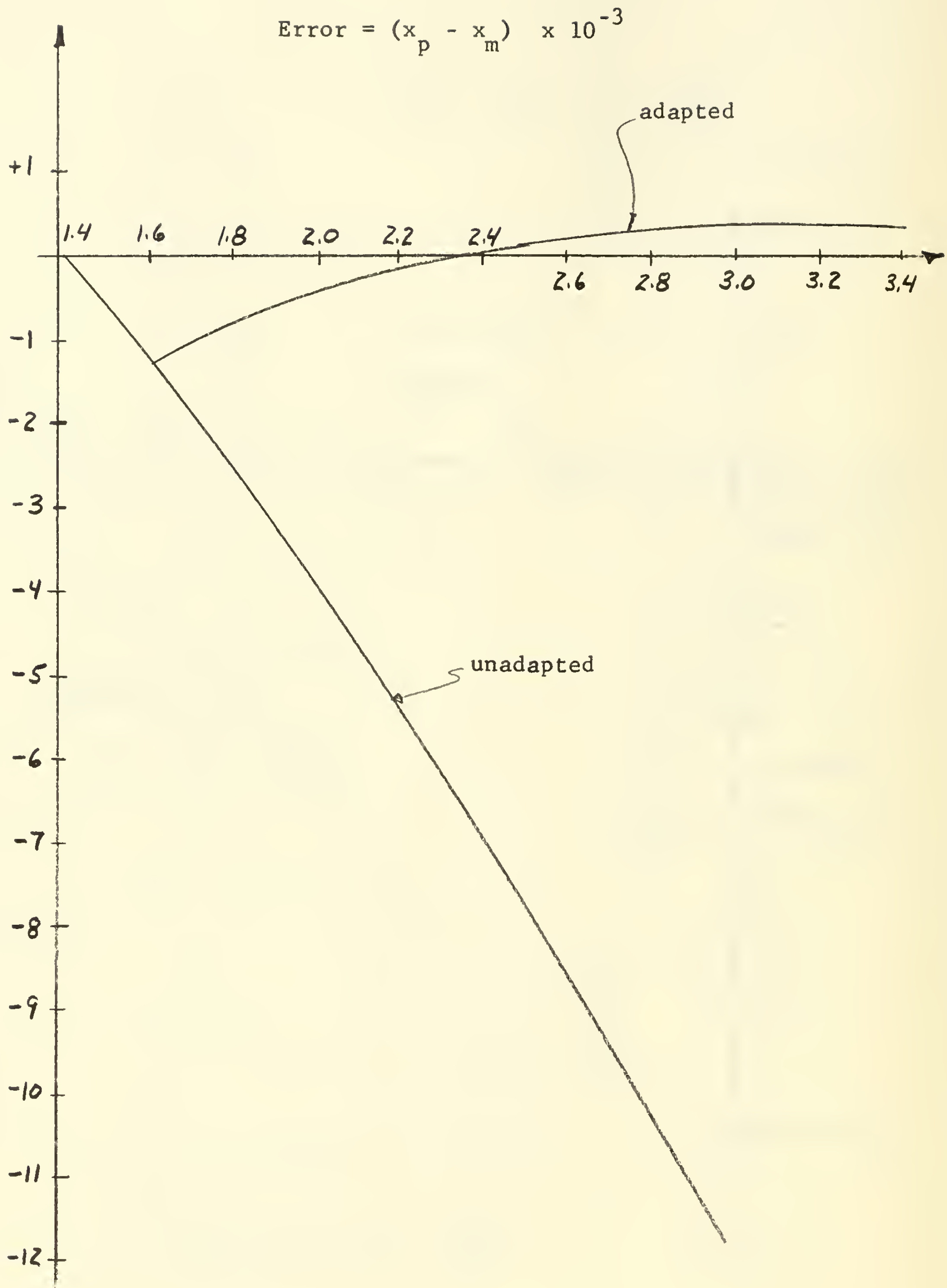
Fig. (18)

$$\text{Error} = (x_p - x_m) \quad \times 10^{-2}$$



$r(t) = 10.0$ for all time

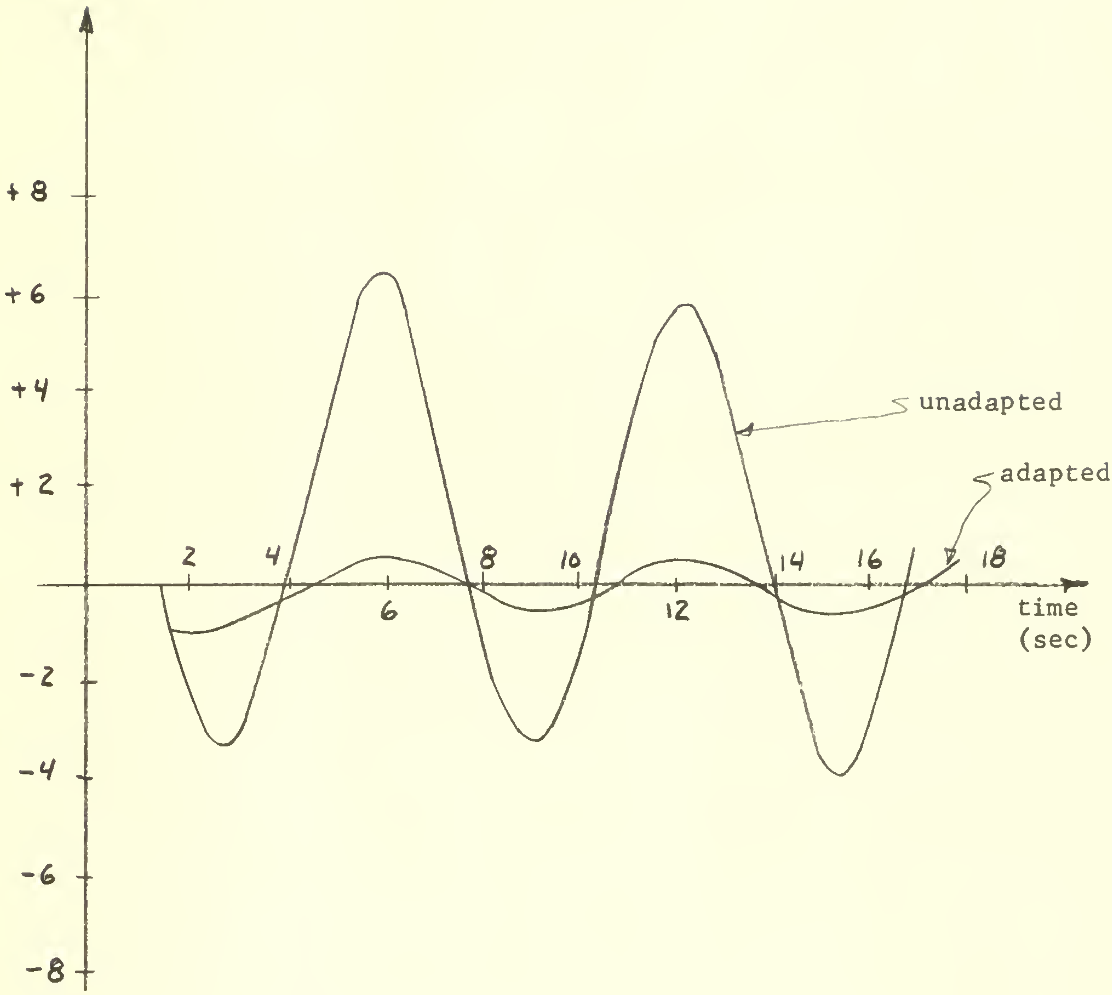
Fig. (19a)



$r(t) = 0.5t$

Fig. (19b)

$$\text{Error} = (x_p - x_m) \quad \times 10^{-2}$$



r(t) = 5.0sin(t)

Fig. (19c)

which yields;

$$u(t) = R - (R - t\,\Delta\alpha - 1)\,\frac{\Delta\alpha}{\alpha}\,e^{-\alpha_0 t} \tag{40}$$

Equation (40) can be simulated and solved by the adaptive controller using the values of the parameters determined by the identification process. Equation (40) represents the exact time relationship for the control to apply to the plant at time t = T.

Example (1) First order plant, one variable parameter

This example serves to illustrate the use of adaptive technique two on the low pass filter of the previous example. The flow graph for the computer simulation of this example is shown in figure (18).

For this example, the variable parameter was changed from 0.2 to 0.19 at time T = 1.5 sec. Parameter identification is accomplished as in Section 5.

The identified value of the parameter is used in the numerical method of steepest descent which generates an optimum control, u*(t), for the time interval $T \leqslant t \leqslant t_f$. This optimum control history is then applied to the plant. The steepest descent method of generating an optimum control is described in Appendix (A).

The results of this example are shown in figures (19a), (19b), and (19c), for step, ramp, and sinusoidal inputs.

One of the inaccuracies introduced into technique two arises from the method of generating the optimum control. To improve the accuracy of the optimum control, the stopping criteria for the steepest descent method may be made smaller, thus increasing the number of iterations required to achieve optimality. Theoretically it would take an infinite number of iterations to reach an exact optimum.

Technique two is less accurate than technique one. However, the

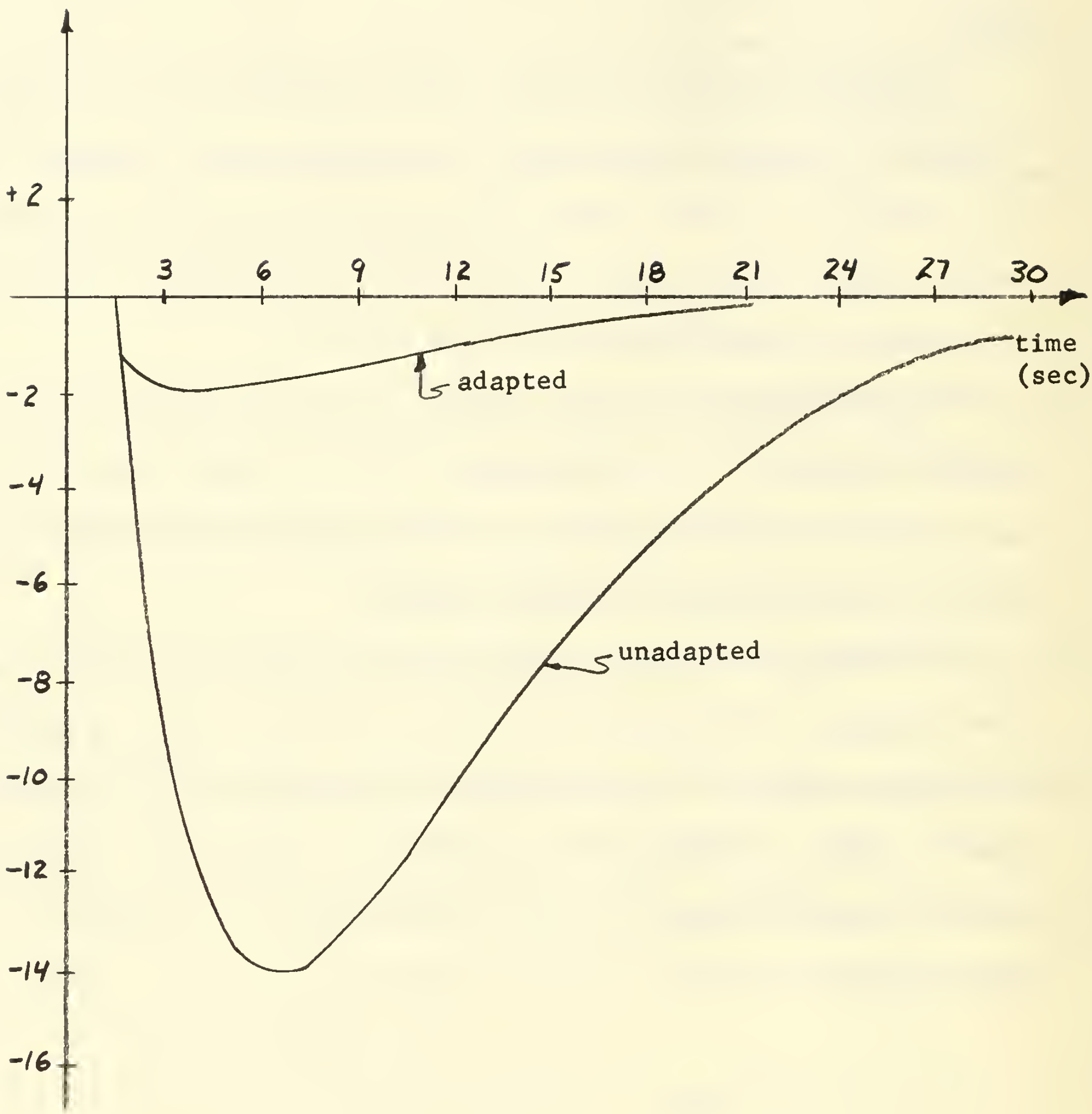
magnitude of the error has, in general, been decreased by at least a factor of ten. An overall improvement in system performance is noticeable.

Equation (40) gives the exact time relationship for the control to be applied in adaptive technique two for this example. Figure (20) shows the resulting error for a step input using equation (40) to generate the control rather than using the method of steepest descent. The identification process provides values for $\Delta\alpha$ and α .

Both the numerical method and the method of equation (40) show a marked improvement in the performance of the system. However, the numerical method is more accurate in that the magnitude of the error is smaller and the error approaches zero more rapidly.

The main disadvantage to the use of a numerical method is that it is time consuming. This is due to the fact that the numerical method is an iterative procedure. The exact time relationship for the control, when used in adaptive technique two, results in a shorter time required for the overall adaptive process. This is a large factor to consider in a real time problem.

Error = $(x_p - x_m) \times 10^{-2}$



r(t) = 10.0 for all time

Fig. (20)

7. Conclusions.

A model-referenced adaptive control system provides an excellent means of compensating for parameter variations caused by changing environmental conditions. This type of adaptive control system requires apriori knowledge of the nominal plant differential equations and will, in general, be implemented by a digital controller. However, it is possible to accomplish the self-adaptive scheme with analog models.

Sensitivity functions provide an excellent tool for the purpose of parameter identification. The parameter identification process is the most critical part of the adaptive scheme in that proper modification can not be accomplished with inaccurate parameter identification. All of the work presented in this paper assumed that the process of measuring the output of the plant was free of measurement noise. When noise is added to the output of the plant, parameter identification becomes a very difficult task. The problem of noise in the output, and the methods which are necessary to overcome the noise in the proposed identification process, are beyond the scope of this thesis.

This paper has suggested three means by which the modification process may be accomplished. Each of these techniques has proven to be very effective.

8. Suggestions for Further Investigation.

As a result of this study there are several areas where further research is recommended.

(a) The effects of observation noise on the identification process.

(b) Identification of several variables which change independently.

(c) Extension of parameter plane techniques for the compensation of uncontrollable parameter variations.

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APPENDIX A

Method of Steepest Descent

The method of steepest descent is a numerical procedure for generating an optimal control history by minimizing a given cost function with respect to the control.

The cost function which is chosen, in this paper, to be minimized is.

(A-1) $$J = \frac{1}{2}\int_{t_0}^{t_f}\left[x_p(t) - x_m(t)\right]^2 dt$$

where $x_p(t) - x_m(t) = e(t)$

The output of the plant is subject to the constraint

$$\dot{x}_p(t) = \alpha x_p(t) + \beta u(t)$$

The Hamiltonian for this cost function is given by,

(A-2) $$\mathcal{H} = x_p^2(t) + x_m^2(t) - 2x_p(t)x_m(t) + p(t)\left[\alpha x_p(t) + \beta u(t)\right]$$

where p(t) is the costate for the above constraint.

The costate equations are given by,

(A-3) $$\dot{p}(t) = \frac{-\partial \mathcal{H}}{\partial x_p(t)}$$

$$\dot{p}(t) = -2\left[x_p(t) - x_m(t)\right] - \alpha p(t)$$

The gradient of the Hamiltonian with respect to the control, u(t), is given by,

(A-4) $$\nabla_u \mathcal{H} = \frac{\partial \mathcal{H}}{\partial u} = \beta p(t)$$

The boundry conditions are given by,

$$p(t_f) = 0.0$$

$$x_p(t_0) = x_{po}$$

(A-5) $$x_m(t_o) = x_{mo}$$

The iterative procedure is begun by solving the state equations for the plant and for the model for the time interval

$$t_o \leq t \leq t_f$$

using the initial control history

$$u(t) = r(t)$$

where

$u(t)$ = input to the plant

$r(t)$ = input to the model

The model state equations are

(A-6) $$\dot{x}_m(t) = \alpha' x_m(t) + \beta' r(t)$$

where α' and β' are known.

The plant state equations are

(A-7) $$\dot{x}_p(t) = \alpha x_p(t) + \beta u(t)$$

where α and β are determined by the parameter identification process.

The state trajectories for the plant and the model are then used to solve the costate equations. The costate trajectory is then used to determine the gradient of the Hamiltonian with respect to the control. If the initial control history, $u(t)$, was the optimal control then

$$\nabla_u \mathcal{H} \equiv 0$$

and the iterative procedure is terminated.

As it is difficult to make $\nabla_u \mathcal{H} \equiv 0$, a suitable stopping criteria must be chosen. The stopping criteria used in this paper is

(A-8) $$\text{Norm} \leq 0.0001$$

where

(A-9) $$\text{Norm} \triangleq \int_{t_0}^{t} (\nabla_u \mathcal{H})^2 dt$$

If the stopping criteria is not satisfied then the control is updated by,

(A-10) $$u(t) = u(t) - \Psi \nabla_u \mathcal{H}$$

and the iterative procedure returns to the state equations. The steepest descent step size, Ψ , must be changed if the cost function is increasing from one iteration to the next, and must be constant if the cost function decreases from one iteration to the next. In this paper, Ψ , was set equal to one for the first iteration. At the end of each iteration, the cost function is evaluated. If the cost function is decreasing, Ψ is held constant, and if the cost function is greater than it was for the last iteration, then Ψ is multiplied by one half.

In order for adaptive technique two to be successful using this method, the stopping criteria must be satisfied. This could take many iterations and could be very time consuming.

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13. ABSTRACT

Plant parameter variations due to environmental changes present a problem in any control system design. In this paper, two adaptive control techniques which compensate for large or small parameter variations are proposed. The variations in the parameters are identified by using the error between the plant output and a fixed model output together with the plant sensitivity functions which are used to identify plant parameter variations. One adaptive technique uses the identified parameter value to physically change the plant parameter and the other adaptive technique uses the identified parameter value to generate a compensating input to the plant. A mathematical model for the simultaneous generation of the desired output and the sensitivity functions is described. Several examples using both techniques are considered.

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